An Analytic Approach for Symplectic Particle Tracking in Complex 3-Dimensional Magnetic Structures

Johannes Bahrdt, HZB / BESSY II, APS-ASD, October, 2011

Outline

- Need for accurate tracking in APPLE II devices
- Symplectic tracking based on generating functions
- Analytic description of arbitrary undulator fields
- Potential applications to other magnet structures
- Analytic equations for dynamic field integrals
- Benefits and limitations of shimming techniques

Permanent Magnet Undulators of BESSY II

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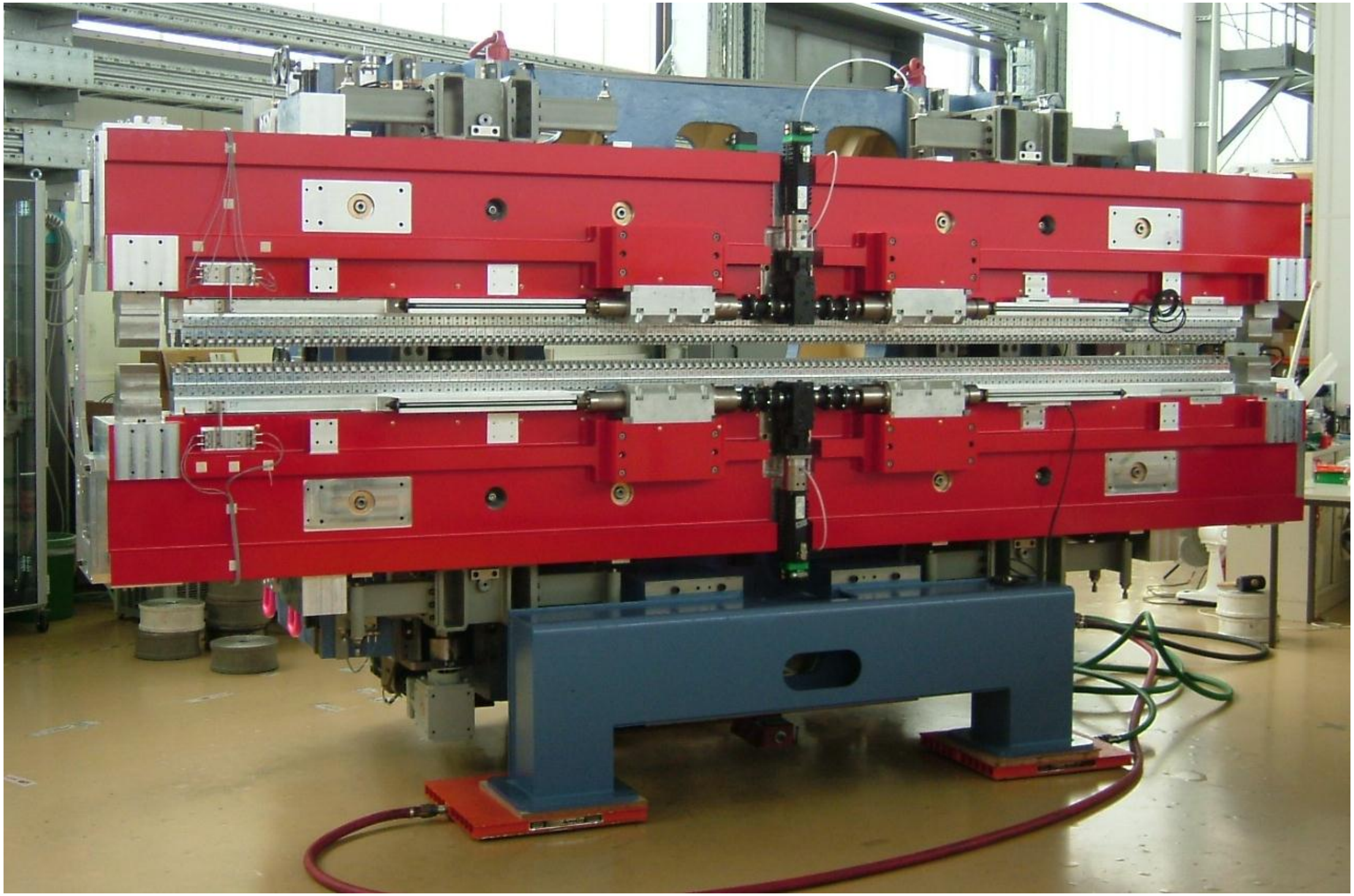
device	design	operational	λ_0 / mm	periods	Gap / mm	By / Bz / T
U49-1	Hybrid	1998 -	49,4	83	15	0,799
U49-2	Hybrid	2000 -	49,4	83	15	0,788
U125-1	Hybrid	1998 – 2005	125	31	20	1,162
U125-2	Hybrid QPU	2000 -	125	31	15	1,360
U41	Hybrid	1999 -	41,2	79	15	0,659
U139	Hybrid	2004 -	139	10	15	1,471
UE56-1	APPLE II	1999 - 2003	56	2 x 30	16	0,771 / 0,529
UE56k	APPLE II	2003 -	56	1 x 30	16	0,772 / 0,529
UE56-2	APPLE II	1999 -	56	2 x 30	16	0,772 / 0,529
UE46	APPLE II	2001 -	46,3	70	16	0,680 / 0,435
UE52	APPLE II	2002 -	52	77	16	0,742 / 0,505
UE49	APPLE II	2003 -	49	63	16	0,709 / 0,477
UE112	APPLE II	2006 -	112	32	20	0,994 / 0,765

UE112 APPLE II Undulator at BESSY II



PETRA III APPLE II Undulator as built at HZB

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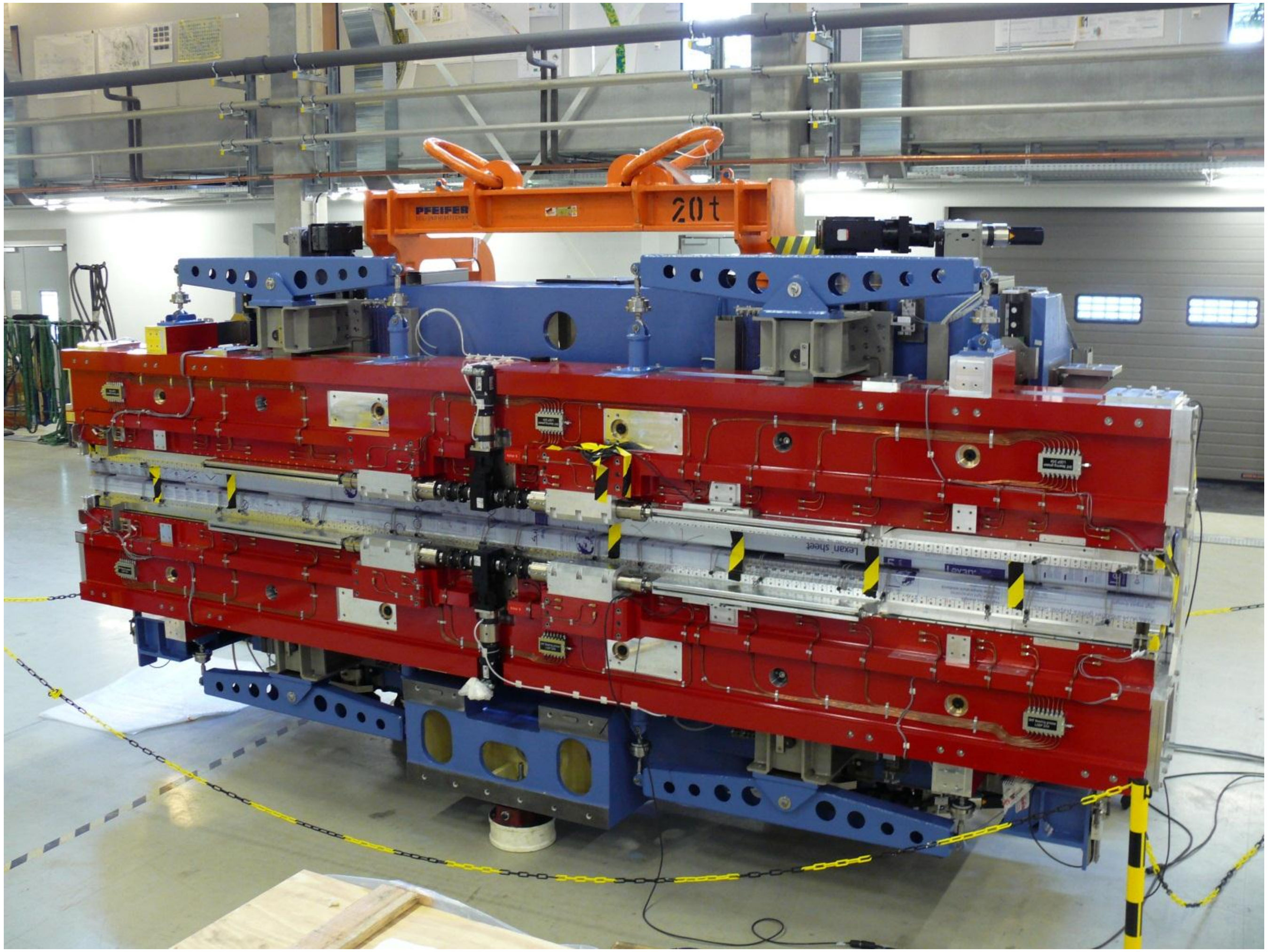
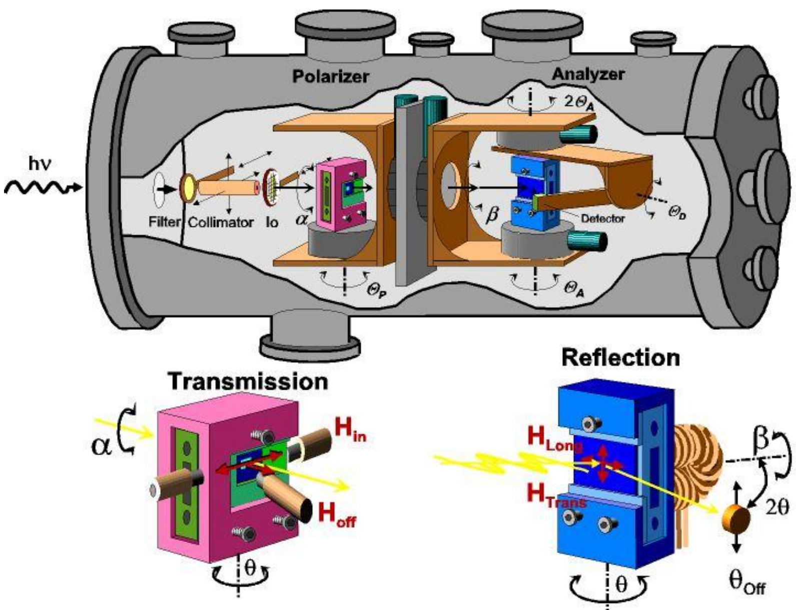
L=5m
gap=11mm

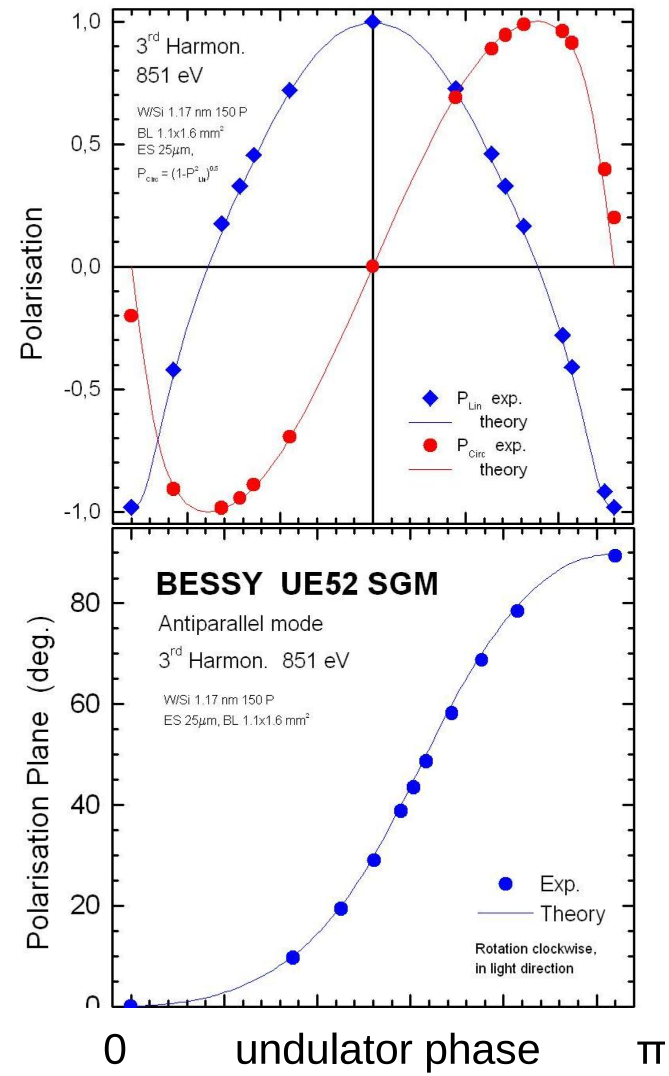


J. Bahrdt et al., Proc. EPAC 2008, Genua, Italy; J. Bahrdt et al., SRI 2009, Melbourne Australia; J. Bahrdt et al., IPAC 2010 Kyoto, Jap; J. Bahrdt et al., IPAC 2011 San Sebastian, Spain



elliptical mode

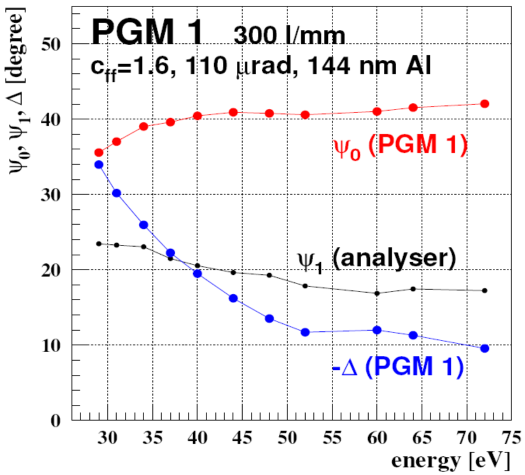
inclined mode



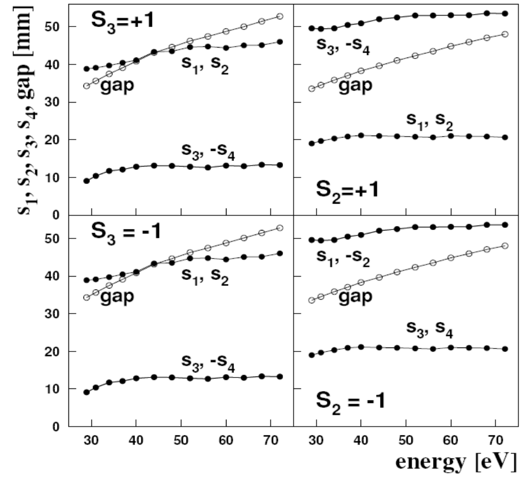
The BESSY Soft X-ray polarimeter

Agreement between theory and measurement

Modification of undulator radiation polarization by beamline components



Parameters of beamline Müller matrix and of analyzer vs. photon energy.



Gap and shift settings for normalized Stokes parameters at the sample.

Müller-Matrix describes beamline effects

$$M = \frac{1}{2}\left(t_x^2 + t_y^2\right)\begin{pmatrix} 1 & \cos 2\psi_o & 0 & 0 \\ \cos 2\psi_o & 1 & 0 & 0 \\ 0 & 0 & \sin 2\psi_o \cos\Delta & \sin 2\psi_o \sin\Delta \\ 0 & 0 & -\sin 2\psi_o \sin\Delta & \sin 2\psi_o \cos\Delta \end{pmatrix}$$

$\tan \psi_o = |t_y|/|t_x|$ transmissions of field amplitudes

$\Delta = \Phi_y - \Phi_x$ phase difference of electric field components

J. Bahrdt et al, SRI 2009, Melbourne, Australia

Need for symplectic tracking code in APPLE II structures

- symplectic and fast particle tracker
- extended undulator structure instead of thin lens approach
- simple interface (30 Fourier coefficients per undulator)
- full parametrization of undulator field in all operation modes (linear superposition of magnet fields)
- simple implementation of shims

dynamic kicks:

$$\theta_{x/y} \sim \lambda_0^2 \cdot B_0^2/\gamma^2$$

Numeric approach

- fast and symplectic, full turn FFAG orbit tracking
  H. Lustfeld, Ph. F. Meads, G. Wüstefeld et al., LINAC 1984
- tracking of superconducting wave length shifter (strong field devices)
  M. Scheer, G. Wüstefeld, EPAC 1992

Analytic approach

- tracking of undulators (nonlinear, weak fields)
  J. Bahrdt, G. Wüstefeld, PAC 1991…
  J. Bahrdt, G. Wüstefeld, Phys. Rev. Special Topics,
  A & B 14, 040703 (2011)

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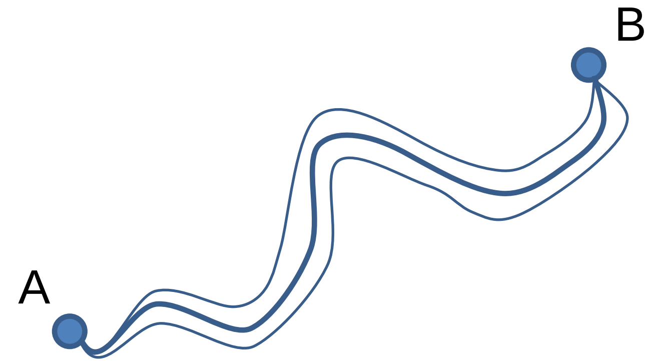
$$\delta \int_A^B L(q,\dot{q})dt = 0$$

Hamilton's principle:
independent variation of coordinates qi

$$\equiv$$

$$\delta \int_A^B (p\dot{q} - H)dt = 0$$

Modified Hamilton's principle:
independent variation of coordinates qi and momenta pi



$$\delta S = 0 = \int_A^B \left(L(q+\varepsilon, \dot{q}+\dot{\varepsilon}) - L(q,\dot{q})\right)dt$$

$$= \int_A^B \left(\varepsilon \frac{\partial L}{\partial q} + \dot{\varepsilon}\frac{\partial L}{\partial \dot{q}}\right)dt$$

integration by parts using $\varepsilon(t_A) = \varepsilon(t_B) = 0$

$$\delta S = \int_A^B \varepsilon(t)\left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\right)dt$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

theorem about the calculus of variation

Newton's Laws

Lagrange Euler equation
2nd order DEs in N variables: $\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$

Hamilton's equations of motion
1st order DEs in 2N variables:

$$\frac{\partial \tilde{H}}{\partial x} = -\dot{p}_x \quad \frac{\partial \tilde{H}}{\partial y} = -\dot{p}_y \quad \frac{\partial \tilde{H}}{\partial p_x} = \dot{x} \quad \frac{\partial \tilde{H}}{\partial p_y} = \dot{y}$$

Differential equations: complete description of system but:
- small step sizes required (pretty nasty for undulators)
- symplectic integrator needed

Alternative method: Integration of Hamilton Jacobi Equation
- permits step sizes as long as undulator length
- symplectic even for long step sizes

Hamiltonian of relativistic particle in magnetic field:

$$\widetilde{H} = \sqrt{(\vec{\tilde{p}} - e\vec{\tilde{A}})^2 c^2 + m_0^2 c^4}$$

Canonical variables: x, y, p_x, p_y, and independent variable t
Hamiltonian is independent upon t.
Change independent variable from time t to z to enable further transformation to cyclic coordinates; new Hamiltonian:

$$\widehat{H} = -\tilde{p}_z = -\sqrt{\frac{\widetilde{H}^2}{c^2} - m_0^2 c^2 - \left(\tilde{p}_x - e\tilde{A}_x\right)^2 - \left(\tilde{p}_y - e\tilde{A}_y\right)^2} - e\tilde{A}_z$$

Normalization and 2nd order expansion in p_x, p_y, x_3

$$H = -p_z = -1 + (p_x - A_x x_3)^2/2 + (p_y - A_y x_3)^2/2 - A_z x_3$$

$x_3 \sim 1/kB\varrho$ small quantity in undulators; well suited as expansion variable

Goal:
Change of canonical variables to cyclic variables using generating functions

Usual procedure of canonical transformation (keeps stationarity of action):

$$p_i\dot{q}_i - H = P_i\dot{Q}_i - K + \frac{dF}{dz}$$

with various options for F

We choose $F = F_3(Q, p, z) + q_i p_i$ getting the new Hamiltonian:

$$K\left(Q_{xi}, Q_{yi}, P_{xi}, P_{yi}\right) = 0 = H\left(x_f, y_f, p_{xf}, p_{yf}\right) + \frac{\partial F_3(Q_{xi}, Q_{yi}, p_{xf}, p_{yf}, z)}{\partial z}$$

I

and the differential equations:

$$P = -\frac{\partial F_3}{\partial Q} \qquad q = -\frac{\partial F_3}{\partial p}$$

Note:

final coordinates and momenta x_f, y_f, p_{xf}, p_{yf} depend upon z

initial coordinates and momenta $Q_{xi}, Q_{yi}, P_{xi}, P_{yi}$ are constants of motion

Analytic Approach for Symplectic Mapping III

With substitutions

$$p_{xf} = -\frac{\partial F_3}{\partial x_f} = -F_{3x} \qquad p_{yf} = -\frac{\partial F_3}{\partial y_f} = -F_{3y}$$

get HJE

$$-1 + (-F_{3x} - A_x x_3)^2 / 2 + (-F_{3y} - A_y x_3)^2 / 2 - A_z x_3 + F_{3z} = 0$$

Insert Taylor series expansion of generating function in HJE

$$F_3 = \sum_{ijk} f_{ijk} p_x^i p_{yf}^j x_3^k$$

expansion variables: p_{xf}, p_{yf}, x_3

Each individual expansion term must be zero.
Iterative solution and determination of z-derivatives of f_{ijk}
Integration of $\partial f_{ijk} / \partial z$ along z yields generating function.

implicit formulation of transformation:

$$x_f = -\partial F_3 / \partial p_{xf}$$
$$y_f = -\partial F_3 / \partial p_{yf}$$
$$p_x = -\partial F_3 / \partial x$$
$$p_y = -\partial F_3 / \partial y$$

A 2nd order expansion of GF includes the following terms

$$f_{001z} = A_z$$
$$f_{011z} = f_{001y} + A_y$$
$$f_{101z} = f_{001x} + A_x$$
$$f_{002z} = -(f_{001x} + A_x)^2/2 - (f_{001y} + A_y)^2/2.$$

Note: the term x_3 always appears (last index ≥ 1)
thus, GF is linear in momenta in 2nd order expansion

Integration of expansion coefficients with respect to z:

$$f_{001} = \int A_z dz$$
$$f_{101} = \int (A_x + \int (\partial A_z/\partial x) dz') dz$$
$$f_{011} = \int (A_y + \int (\partial A_z/\partial y) dz') dz$$
$$f_{002} = -(1/2)\cdot \int \left((A_x + \int (\partial A_z/\partial x) dz')^2 + (A_y + \int (\partial A_z/\partial y) dz')^2 \right) dz.$$

From the generating function the transformation map is evaluated

Transformation Map

2nd order means 1st order in momenta:

$$
\begin{aligned}
x_f &= x + p_{xf} z - f_{101} \\
p_x &= p_{xf} - f_{101x} p_{xf} - f_{011x} p_{yf} - f_{002x} - f_{001x} \\
y_f &= y + p_{yf} z - f_{011} \\
p_y &= p_{yf} - f_{101y} p_{xf} - f_{011y} p_{yf} - f_{002y} - f_{001y}.
\end{aligned}
$$

In this case the implicit form can be converted into an explicit form:

$$
\begin{aligned}
p_{xf} &= ((1 - f_{011y})(p_x + f_{002x} + f_{001x}) + f_{011x}(p_y + f_{002y} + f_{001y})) / p_n \\
x_f &= x - f_{101} + p_{xf} z_f \\
p_{yf} &= ((1 - f_{101x})(p_y + f_{002y} + f_{001y}) + f_{101y}(p_x + f_{002x} + f_{001x})) / p_n \\
y_f &= y - f_{011} + p_{yf} z_f .
\end{aligned}
$$

With $p_n = (1-f_{011y})(1-f_{101x})-f_{011x}f_{101y}$

Note: Transformation is not limited to 2nd order, however, the Newton Raphson Method has to be used for higher order expansions to solve the implicit equations

Scalar potential of a Halbach type undulator:

$$V = -(B_0 / k_y)\cos(k_x x)\sinh(k_y y)\cos(kz+\varphi).$$

Derivation of vector potential from scalar potential

$$A_x = -\int (\partial V / \partial y) dz + C_1$$

$$A_y = \int (\partial V / \partial x) dz + C_2$$

$$A_z = 0.$$

which is

$$A_x = (B_0 / k_z)\cos(k_x x)\cosh(k_y y)\sin(kz+\varphi)$$

$$A_y = ((B_0 k_x)/(k_y k_z))\sin(k_x x)\sinh(k_y y)\sin(kz+\varphi)$$

$$A_z = 0.$$

From this expressions the particle dynamics in undulators can be derived

Note:
Analytic expressions of the vector potential are required since GF is derived from analytic integration over z

Example II: Arbitrary Periodic Structure in x and z

The following functions $\cos(k_{xn}x)\cos(k_{zm}z)$, $\sin(k_{xn}x)\cos(k_{zm}z)$
$\cos(k_{xn}x)\sin(k_{zm}z)$, $\sin(k_{xn}x)\sin(k_{zm}z)$

form a basis on the interval $S=[-\lambda_{x0}/2, \lambda_{x0}/2]\text{x}[-\lambda_{z0}/2, \lambda_{z0}/2]$
for all periodic functions in x and z direction with periodicity λ_{x0} *and* λ_{z0}

Complete description of arbitrary periodic magnet fields in a plane:

$$B_y(x,z) = \sum_{i=0}^{n}\sum_{j=0}^{m}(cc_{i,j}\cos(k_{xi}x)\cos(k_{zj}z) + sc_{i,j}\sin(k_{xi}x)\cos(k_{zj}z) +$$

$$cs_{i,j}\cos(k_{xi}x)\sin(k_{zj}z) + ss_{i,j}\sin(k_{xi}x)\sin(k_{zj}z))$$

$$k_{zj} = jk_{z1} = j2\pi/\lambda_{z0}$$

$$k_{xi} = ik_{x1} = i2\pi/\lambda_{x0}$$

Fourier coefficients: $cc_{i,j}, cs_{i,j}, sc_{i,j}, ss_{i,j}$

Ansatz for 3D field: $\widehat{B}_{y,ij}(x,y,z) = B_{y,ij}(x,z)\cdot H_{ij}(y)$

with $\nabla B = \nabla \times B = 0$: $(k_{xi}^2 + k_{\tilde{j}}^2)H = \partial^2 H_{ij}/\partial y^2$

solving the differential equation: $H_{ij} = c_1\exp(-k_{yi,\tilde{j}}y) + c_2\exp(+k_{yi,\tilde{j}}y)$

with midplane symmetry: $H_{ij} = c\cdot\cosh(k_{yi,\tilde{j}}y)$

Skip sin-terms for planar undulator (symmetric in x) and get usual Halbach fields:

$$\widehat{B}_y(x,y,z) = \sum_{i=0}^{n}\sum_{j=1}^{m} c_{i,\tilde{j}} \cos(k_{xi}x)\cosh(k_{yi,\tilde{j}}y)\cos(k_{\tilde{j}}z)$$

$$k_{yi,\tilde{j}} = \sqrt{k_{\tilde{j}}^2 + k_{xi}^2}$$

$$\tilde{j} = 1,3,5\ldots$$

(1,5,9… for ppm structure)

Derivation of Fourier coeffifficients

- undulator:
  single trans. field distribution
- wiggler:
  seveveral trans. field distributions

$$C_{0,1} = \sum_{j=1}^{m} c_{0,\tilde{j}}$$

…

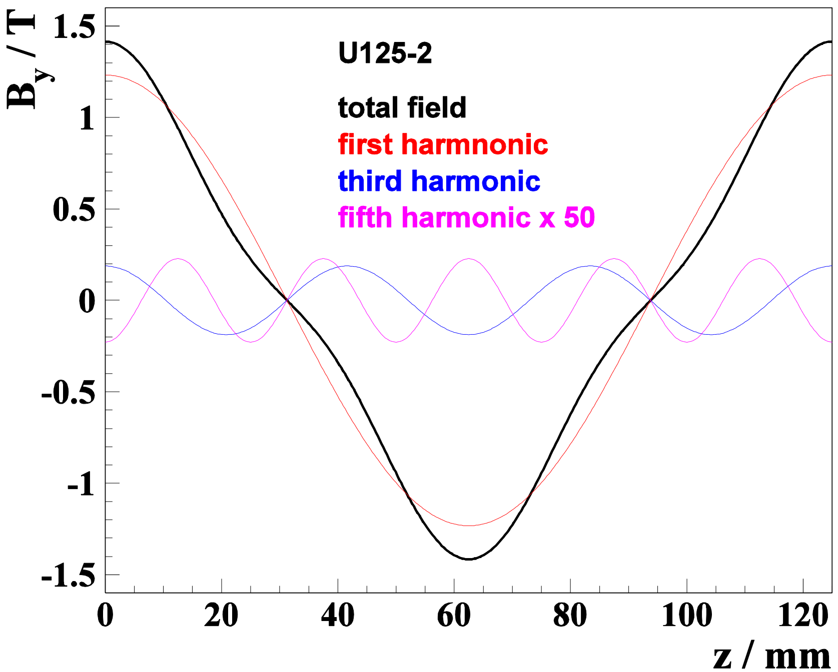
$$C_{n,1} = \sum_{j=1}^{m} c_{n,\tilde{j}}$$

…

$$C_{n,m} = \sum_{j=1}^{m} c_{n,\tilde{j}} \cos(k_{\tilde{j}} z_m)$$

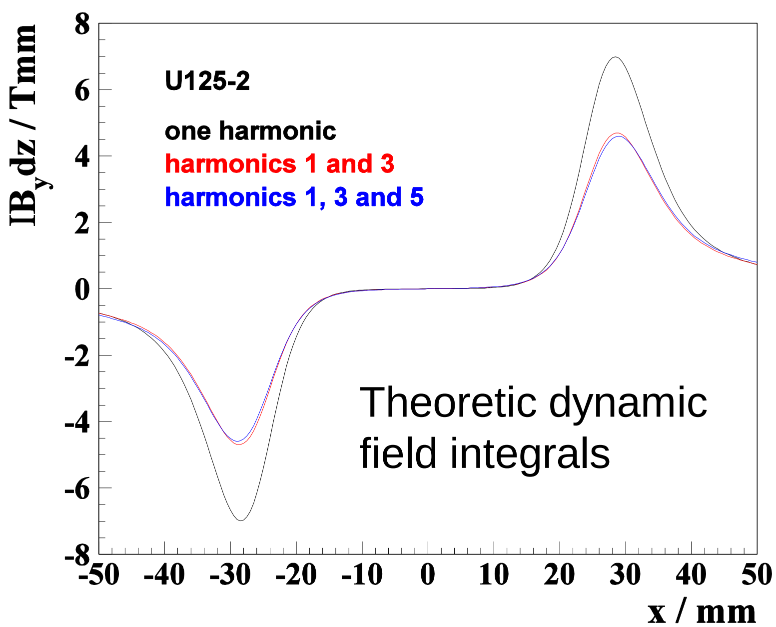
solve system of linear equations

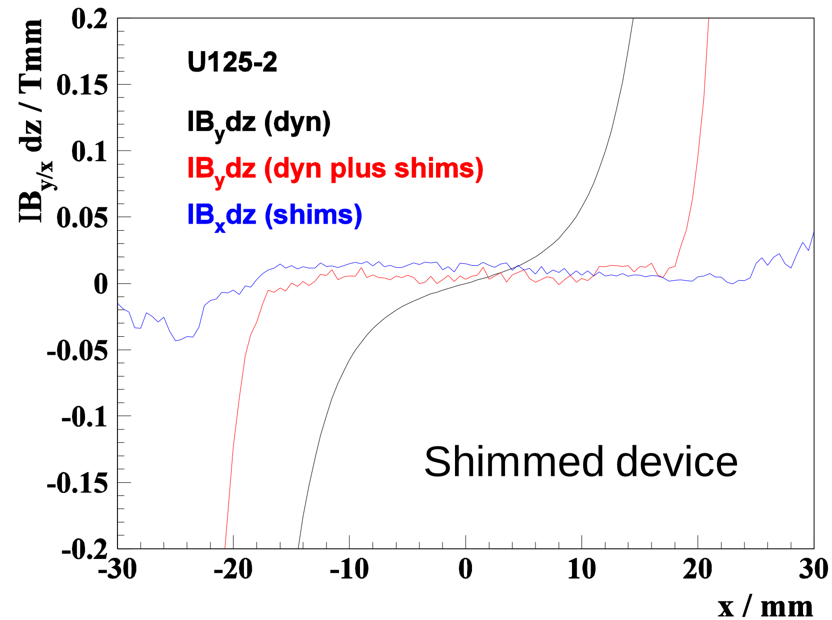
J. Bahrdt et al., IPAC 2011



Field reconstruction using different numbers of harmonics

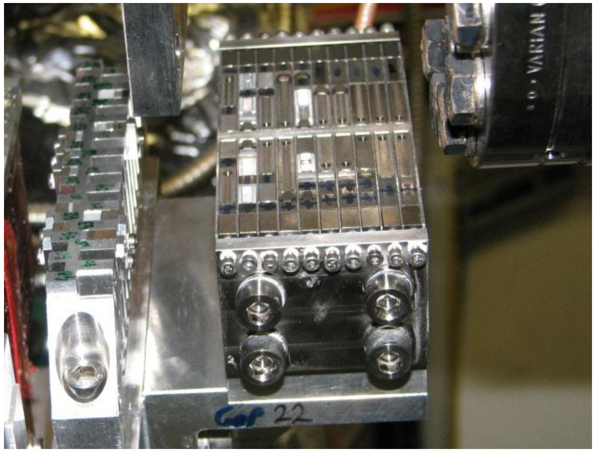
BESSY U125 Wiggler II





The 3rd field harmonic has to be included for a a field reconstruction on the percent level

For undulators usually 1st field harmonic is sufficient



U125-2 Magic fingers

Scalar potential of APPLE II field

$$\vec{B} = -\vec{\nabla}(V)$$

$$V = \sum_{i=1}^{n} (V_{1i} + V_{2i} + V_{3i} + V_{4i})$$

$$V_{1i} = +((e^{+k_{yi}y} / k_{yi}) \cdot (B_{cyi} c_{xi-} + B_{syi} s_{xi-}) + B_0 e^{+k_z y} / nk_z) \cdot c_{z+}$$

$$V_{2i} = +((e^{+k_{yi}y} / k_{yi}) \cdot (B_{cyi} c_{xi+} + B_{syi} s_{xi+}) + B_0 e^{+k_z y} / nk_z) \cdot c_{z-}$$

$$V_{3i} = -((e^{-k_{yi}y} / k_{yi}) \cdot (B_{cyi} c_{xi+} + B_{syi} s_{xi+}) + B_0 e^{-k_z y} / nk_z) \cdot c_{z+}$$

$$V_{4i} = -((e^{-k_{yi}y} / k_{yi}) \cdot (B_{cyi} c_{xi-} + B_{syi} s_{xi-}) + B_0 e^{-k_z y} / nk_z) \cdot c_{z-}$$

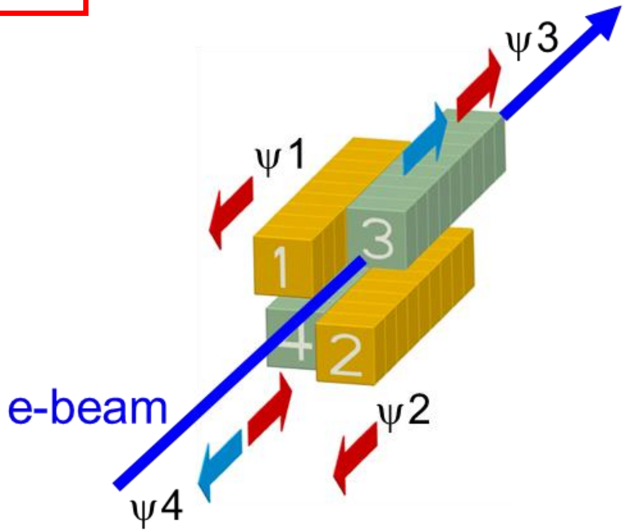
$$c_{xi\pm} = \cos(k_{xi}(x \pm x_0))$$

$$s_{xi\pm} = \sin(k_{xi}(x \pm x_0))$$

$$c_{z\pm} = \cos(k_z z \pm \psi / 2)$$

$$k_{xi} = i \cdot k_{x0}$$

$$k_{yi} = \sqrt{k_{xi}{}^2 + k_z^2}$$



helical undulator
shifting of the poles

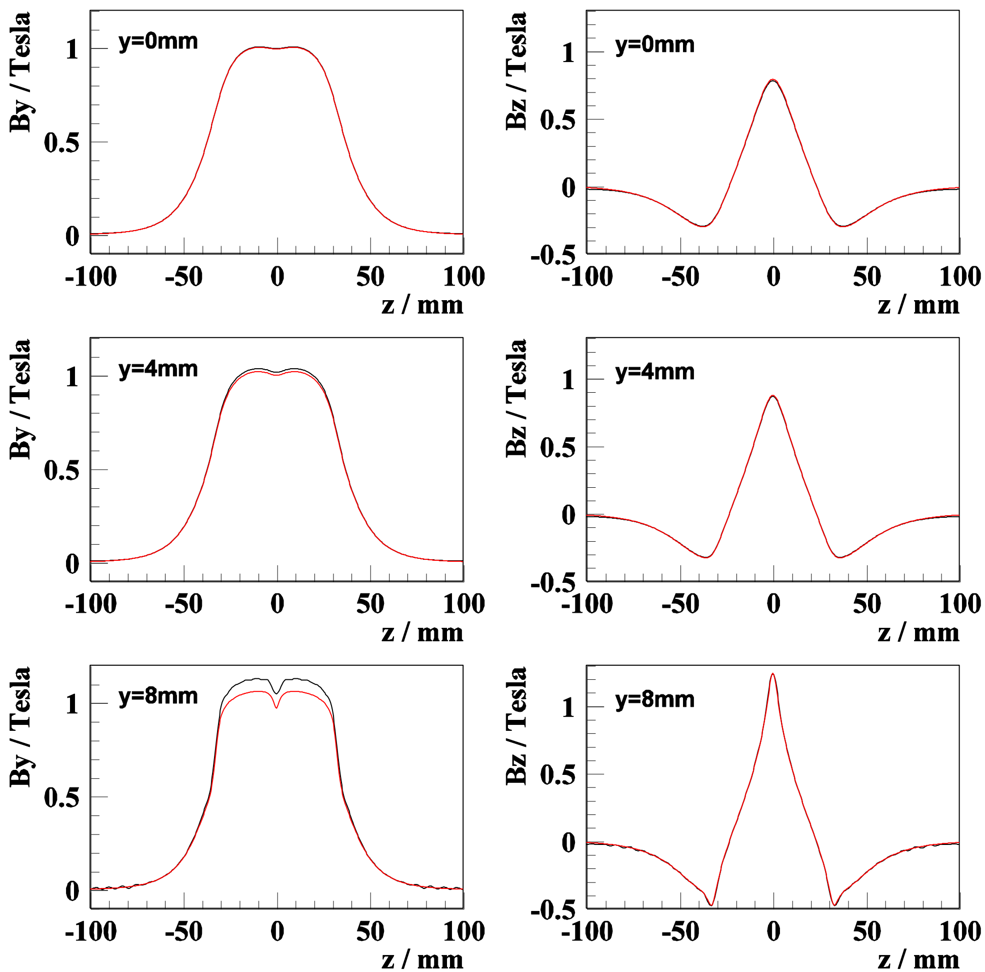
$$A_x = -\int (\partial V / \partial y) dz + C_1$$

$$A_y = \int (\partial V / \partial x) dz + C_2$$

$$A_z = 0.$$

Accuracy of APPLE II Field Description

By at phase = 0 | Bx at phase = pi



analytic representation (black)
simulation with RADIA (red)

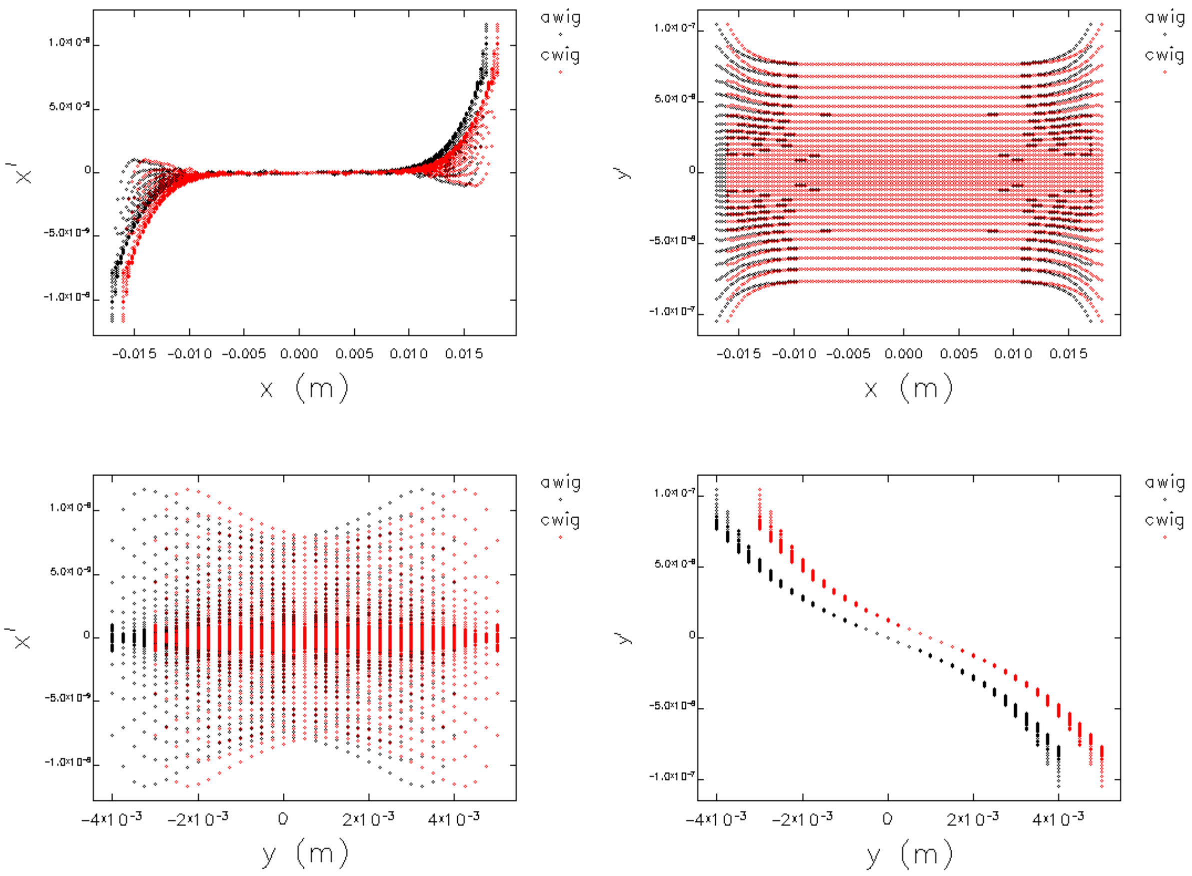
in midplane

4mm vertical off-axis

8mm vertical off-axis

⇒ Parametrization of magnet field distribution with high accuracy

Comparison with ELEGANT



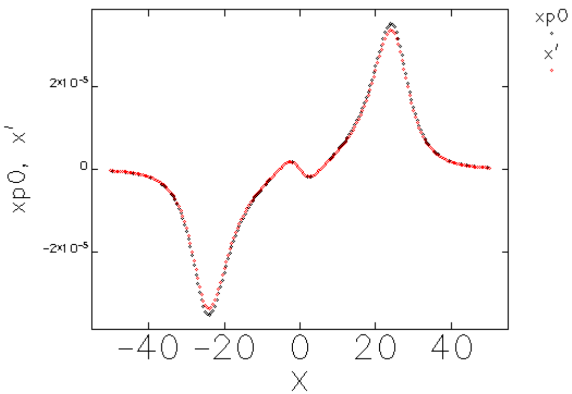
Planar undulator
Period length 28mm

Analytic approach and symplectic tracker cwigg of ELEGANT

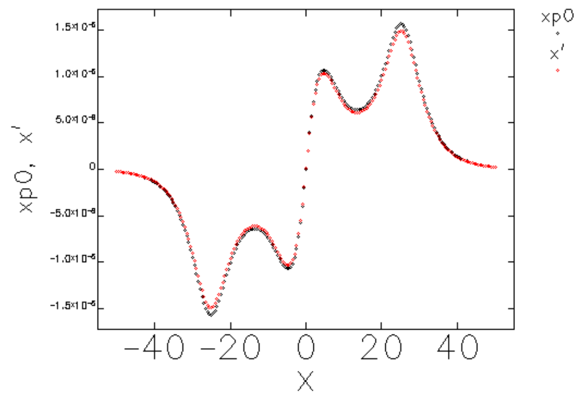
Aimin Xiao

Acceleration of undulator tracking
By more than an order of magnitude

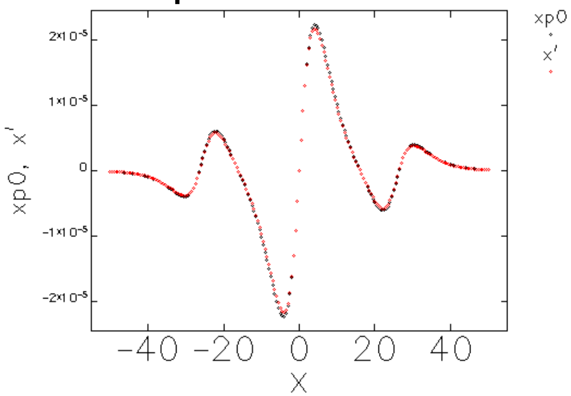
First Results for APPLE II



phase = 0



phase = pi/2



phase = pi

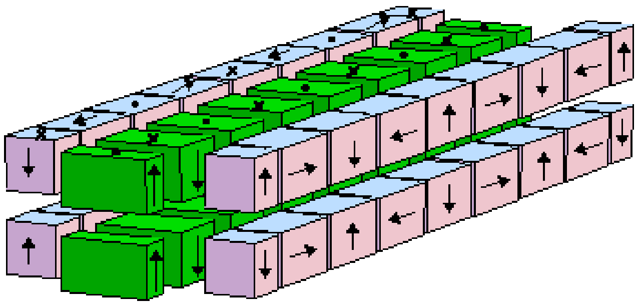
Aimin Xiao

Aimin‘s implementation of analytic method into ELEGANT

Comparison of thick lens and thin lens approach

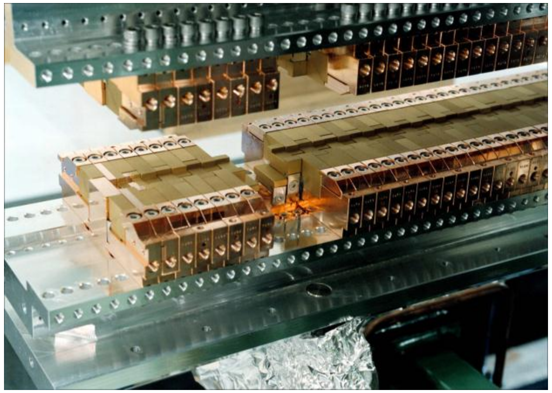
- Other complicated undulator structures such as

Figure 8 undulator for lin. pol.
and reduced on axis power density



Courtesy of B. Diviacco

SPRING 8 helical ID

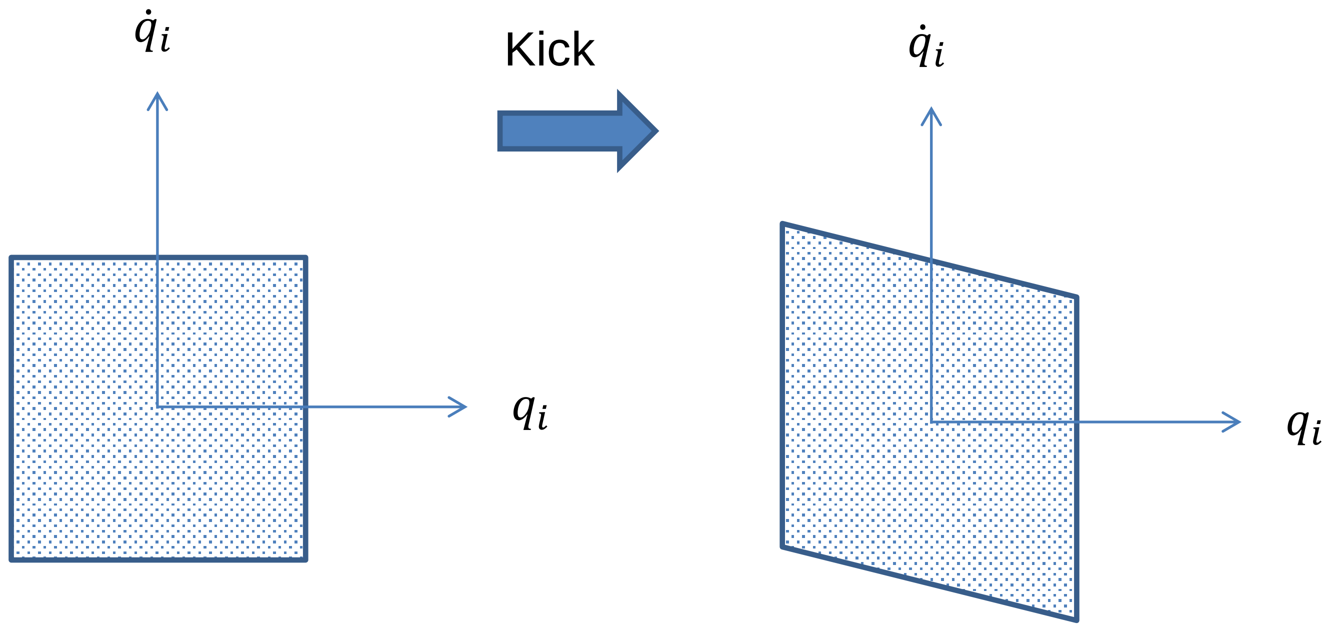


Courtesy of H. Kitamura

- Dipole magnet:
  Curved coordinate system along the reference orbit is used
  particle displacements and momenta with resect to this reference orbit
  starting from a modified Hamiltonian p_s the same method is applicable

- Fringe fields in quadrupoles and sextupoles

- …



$$\dot{q}_i{}^{new} = \dot{q}_i{}^{old} + W(q_i)$$

phase space volume is preserved with kick maps

The integrated dynamic kicks due to the wiggling motion in undulators follow directly from generating function:

$$\theta_x = \partial f_{002} / \partial x \qquad \theta_y = \partial f_{002} / \partial y$$

$$\theta_x = -\frac{z_f}{2(B\rho)^2} \sum_{i1=0}^{n} \sum_{i2=0}^{n} \sum_{j=1}^{m} c_{i1,\tilde{j}} c_{i2,\tilde{j}} \frac{k_{xi1}}{k_{\tilde{j}}^2} \sin(k_{xi1}x)\cos(k_{xi2}x)$$
$$\times \left(\frac{1}{k_{yi1,\tilde{j}}} \frac{k_{xi2}^2}{k_{yi2,\tilde{j}}} \sinh(k_{i1,\tilde{j}}y)\sinh(k_{i2,\tilde{j}}y) - \cosh(k_{i1,\tilde{j}}y)\cosh(k_{i2,\tilde{j}}y)\right)$$

$$\theta_y = -\frac{z_f}{2(B\rho)^2} \sum_{i1=0}^{n} \sum_{i2=0}^{n} \sum_{j=1}^{m} c_{i1,\tilde{j}} c_{i2,\tilde{j}} \frac{1}{k_{\tilde{j}}^2} \cosh(k_{yi1,\tilde{j}}y)\sinh(k_{yi2,\tilde{j}}y)$$
$$\times \left(k_{yi2,\tilde{j}} \cos(k_{xi1}x)\cos(k_{xi2}x) + k_{xi1} \frac{k_{xi2}}{k_{yi2,\tilde{j}}} \sin(k_{xi1}x)\sin(k_{xi2}x)\right)$$

These expressions can be used in a thin lens approximation (kick map) which assumes a localized kick of the strength of the integrated quantity

Dynamic Kicks of APPLE II Undulators

Elliptical mode ($\varphi_2=0$)

c_{pp}, s_{pp} analytic functions of φ_1

e_i, e_j include gap dependence

$$\theta_x = f_0 \cdot c_{pp}^2 + f_\pi \cdot s_{pp}^2$$

$$\theta_y = 0$$

$$f_0 = \frac{L \cdot 8}{k^2 (B\rho)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j e_i e_j k_{xj} c_{xi0} c_{xi} c_{xj0} s_{xj}$$

$$f_\pi = \frac{L \cdot 8}{k^2 (B\rho)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j e_i e_j \frac{k_{xi}}{k_{yi}} \frac{k_{xj}^2}{k_{yj}} s_{xi0} c_{xi} s_{xj0} s_{xj}.$$

Inclined mode ($\varphi_1=0$):

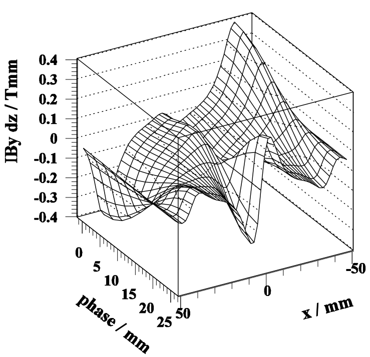
c_{pp}, s_{pp} analytic functions of φ_2

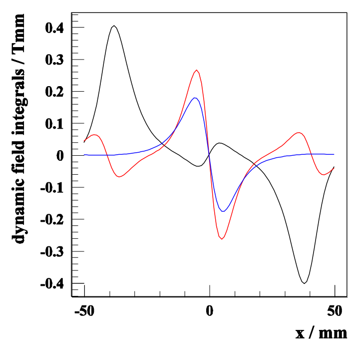
$$\theta_x = f_0 \cdot c_{pa}^4 + f_\pi \cdot s_{pa}^4 + f_{\pi/2} \cdot c_{pa}^2 \cdot s_{pa}^2$$

$$\theta_y = g_{\pi/2} \cdot c_{pa}^2 \cdot s_{pa}^2$$

$$f_{\pi/2} = -\frac{L \cdot 8}{k^2 (B\rho)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j e_i e_j \left(\frac{k_{xi}}{k_{yi}} \frac{k_{xj}^2}{k_{yj}} c_{xi0} s_{xi} c_{xj0} c_{xj} + k_{xj} s_{xi0} s_{xi} s_{xj0} c_{xj} \right)$$

$$g_{\pi/2} = -\frac{L \cdot 8}{k^2 (B\rho)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j e_i e_j \left(\frac{k_{xi}}{k_{yi}} k_{xj} (s_{xi0} c_{xi} c_{xj0} s_{xj} - c_{xi0} s_{xi} s_{xj0} c_{xj}) + k_{yj} \cdot (-c_{xi0} c_{xi} s_{xj0} s_{xj} + s_{xi0} s_{xi} c_{xj0} c_{xj}) \right).$$





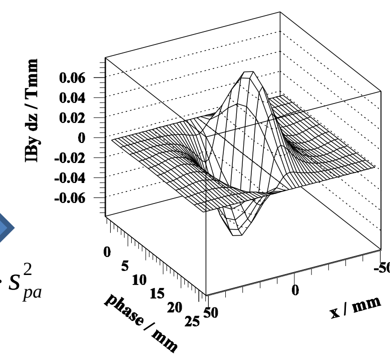
4 generic functions

f_0

f_π

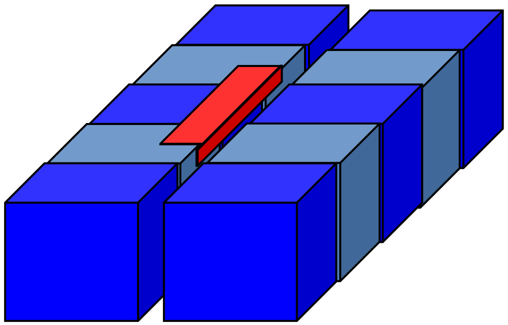
$f_{\pi/2}$

$g_{\pi/2} \cdot c_{pa}^2 \cdot s_{pa}^2$



For general expressions in universal mode and for off-midplane expressions see

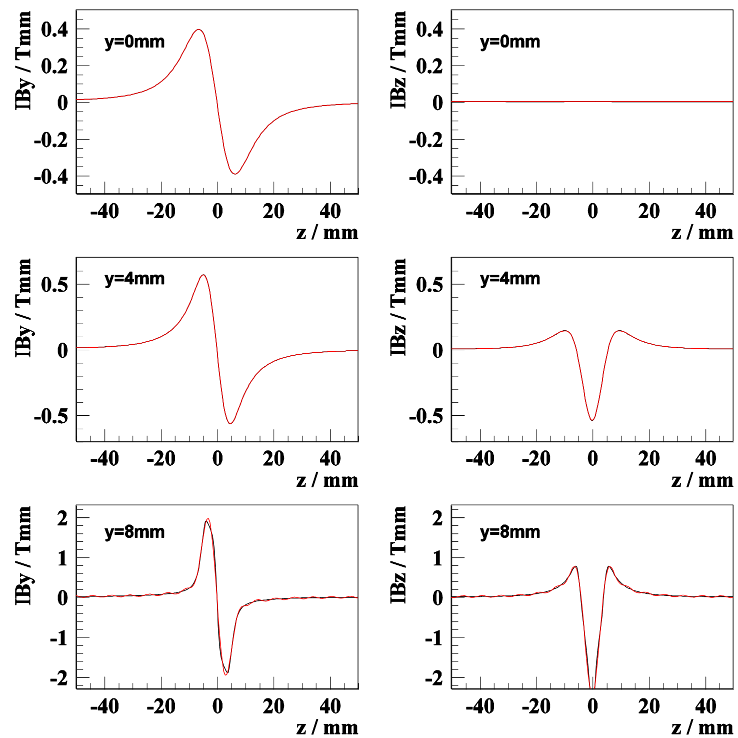
J. Bahrdt, G. Wüstefeld, Phys. Rev. Special Topics, A & B 14, 040703 (2011)



J. Chavanne et al., Proceedings of the EPAC 2000, Vienna, Austria

Field integrals of L-Shim

analytic representation (black)
simulation with RADIA (red)



Implementation in our tracking scheme:

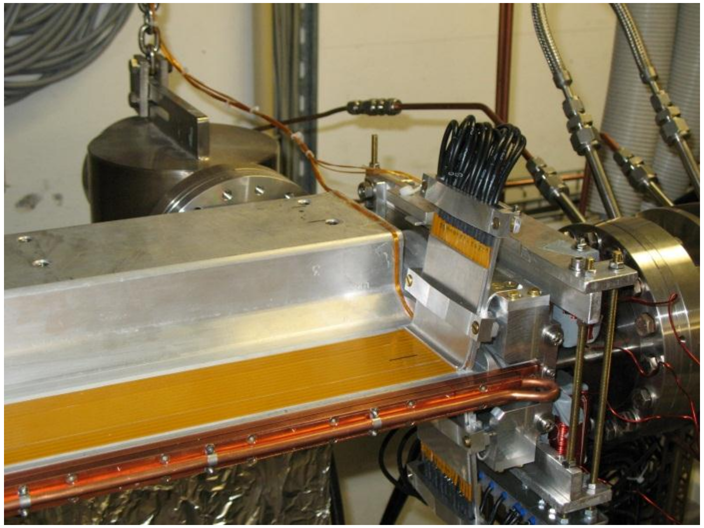
$$\overline{B}_x = \sum_{i=0}^{n} \frac{k_{xi}}{k_{yi}} \left[\overline{c}_i \cdot \sin(k_{xi}x) \cdot \cosh(k_{yi}y) + \overline{s}_i \cdot \cos(k_{xi}x) \cdot \sinh(k_{yi}y) \right] \cdot \exp[-k_{yi}\Delta g / 2]$$

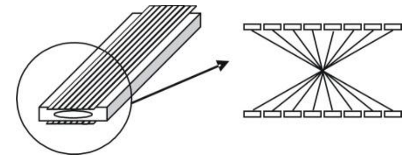
$$\overline{B}_y = \sum_{i=0}^{n} \left[-\overline{c}_i \cdot \cos(k_{xi}x) \cdot \sinh(k_{yi}y) + \overline{s}_i \cdot \sin(k_{xi}x) \cdot \cosh(k_{yi}y) \right] \cdot \exp[-k_{yi}\Delta g / 2]$$

$$k_{xi} = k_{yi}.$$

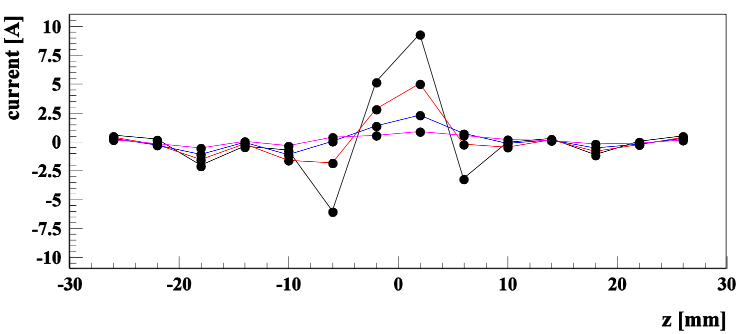
⇒ Parametrization of shim field integral with high accuracy

Active Compensation of Dynamic Kicks with Flat Wires: BESSY II UE112 APPLE II

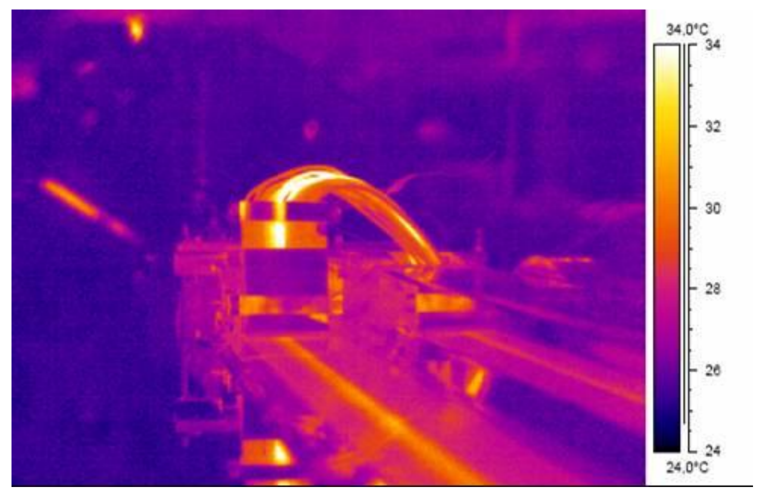




water cooled undulator vacuum chamber



current settings for gaps of 20mm 24mm, 30mm and 40mm



J. Bahrdt et. al., EPAC 2008, Genoa, Italy

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$$\overline{B}_y = -\sum_{m=0}^{\infty} x^m \cdot d \cdot \Big(\sum_{i=istart}^{m+1} \frac{c_{ij}}{r_0^{2i}} \cdot x_0^{2(\lceil m/2 \rceil + i - m - 1) - \mathrm{mod}(m,2)+1} -$$

$$h(m) \cdot \sum_{i=istart-\mathrm{mod}(m,2)}^{m} \frac{c_{ij_1}}{r_0^{2i}} \cdot x_0^{2(\lceil (m-1)/2 \rceil + i - m) - \mathrm{mod}((m-1),2)} \Big)$$

$$\overline{B}_x = \sum_{m=0}^{\infty} x^m \cdot d \cdot y_0 \cdot \sum_{i=istart}^{m+1} c_{ij} \cdot \frac{1}{r_0^{2i}} x_0^{2(\lceil m/2 \rceil + i - m - 1) - \mathrm{mod}(m,2)},$$

$$istart = \lceil m/2 \rceil + 1$$

$$r_0^2 = x_0^2 + y_0^2$$

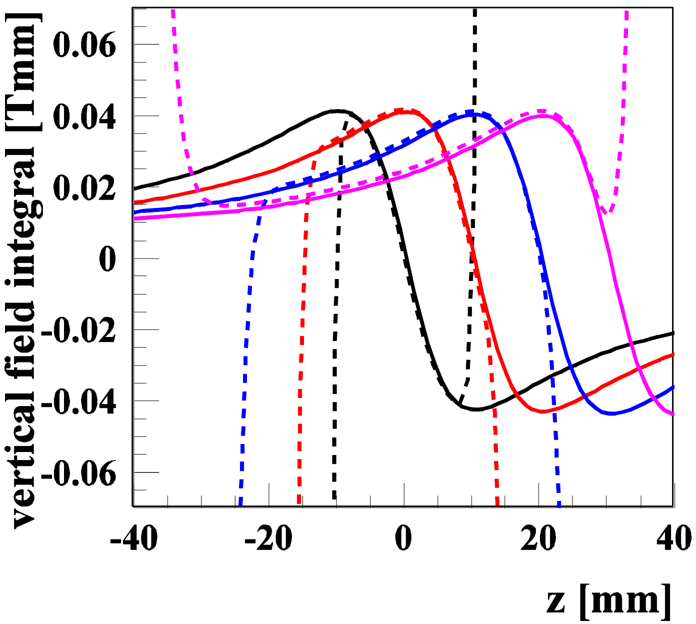
$$j = \mathrm{mod}(m,2) + 2(i - istart) + 1$$

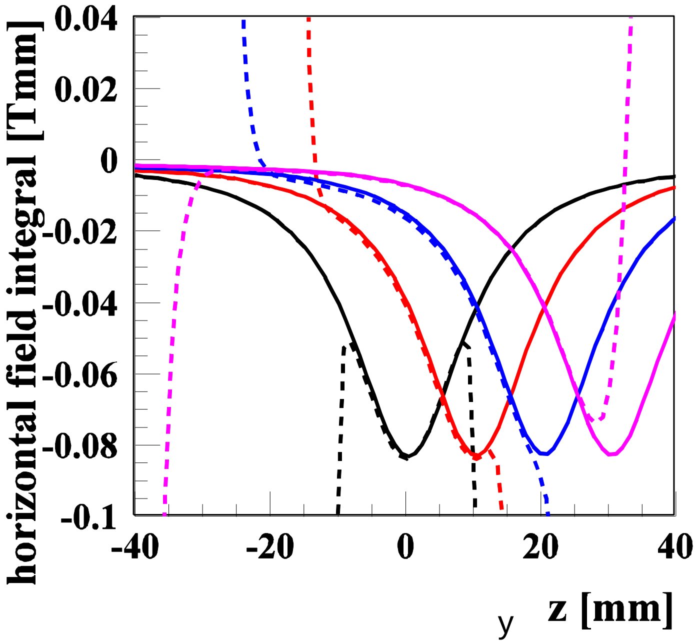
$$j_1 = \mathrm{mod}(m,2) + 2(i - istart) + 2$$

$$c_{ij_{(1)}} = \begin{pmatrix} i-1 \\ j_{(1)} - 1 \end{pmatrix} \cdot (-1)^{i + j_{(1)} - 2} \cdot 2^{j_{(1)} - 1}$$

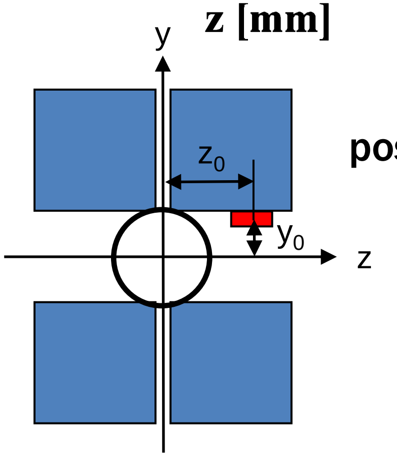
$$h(m) = 1 - \delta_{m,0}.$$

Accuracy of Multipole Expansion





Accuracy of field integral description (y=0) with z_0=0,, 10, 20, 30mm and y_0=10mm. Thick lines: exact fields integrals, dotted lines: approximation with m=20.



➡ multipole expansions is senseless outside radius of convergence
(defined by magnet field sources)
use instead: multipole specs at various hor. trans. positions

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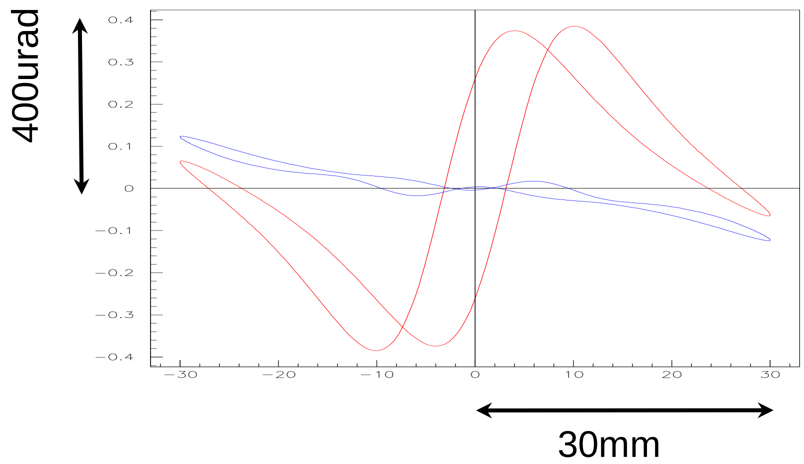
UE112 shimming effect

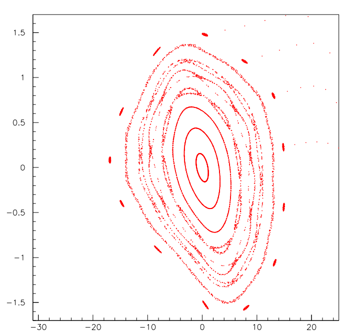


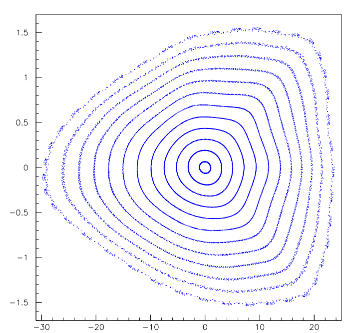
x-kick per ID passage (vertical linear mode)
particles distributed on horizontal phase space ellipse, semi axes: 30mm / 1.87mrad

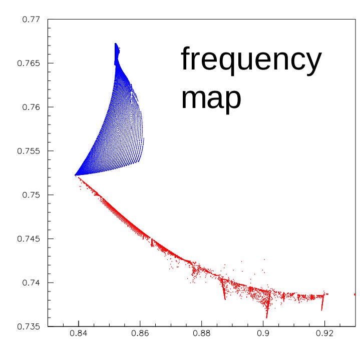
red: no shims

blue: wire shims powered

BESSY II: 1000-turns tracking (x-x'-plane)

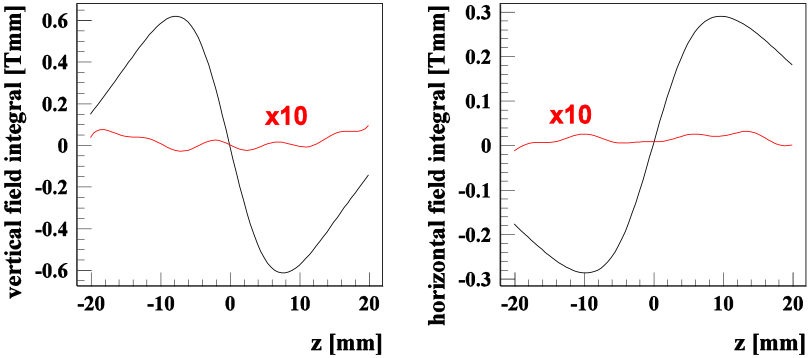




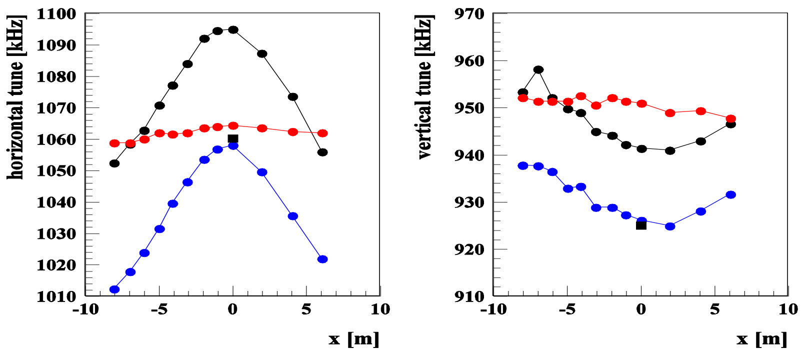


EPAC 2006 Bahrdt, Scheer& Wüstefeld

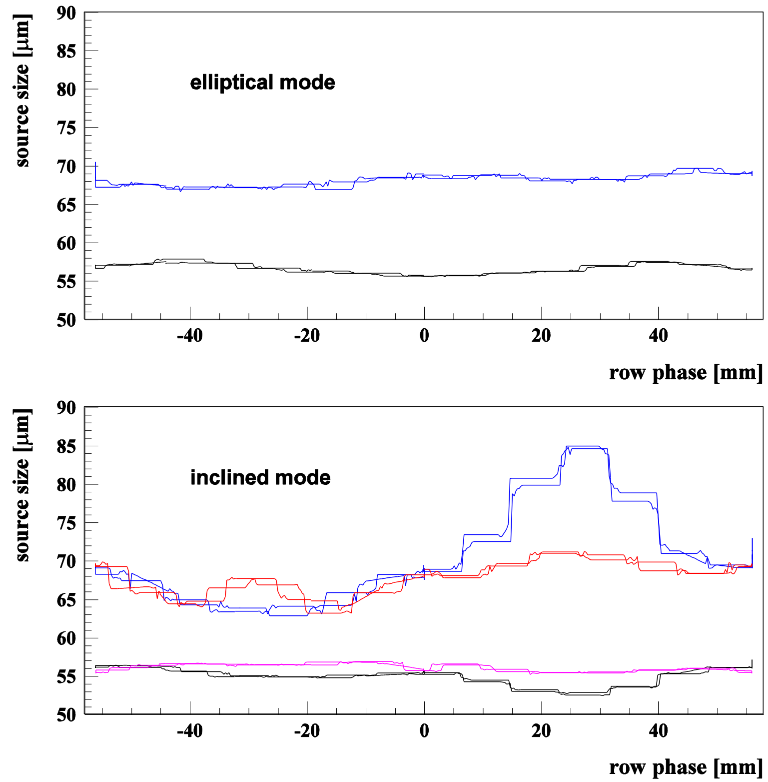
UE112: Tune Shift and Beam Size Variation



dynamic multipoles with and without active compensation

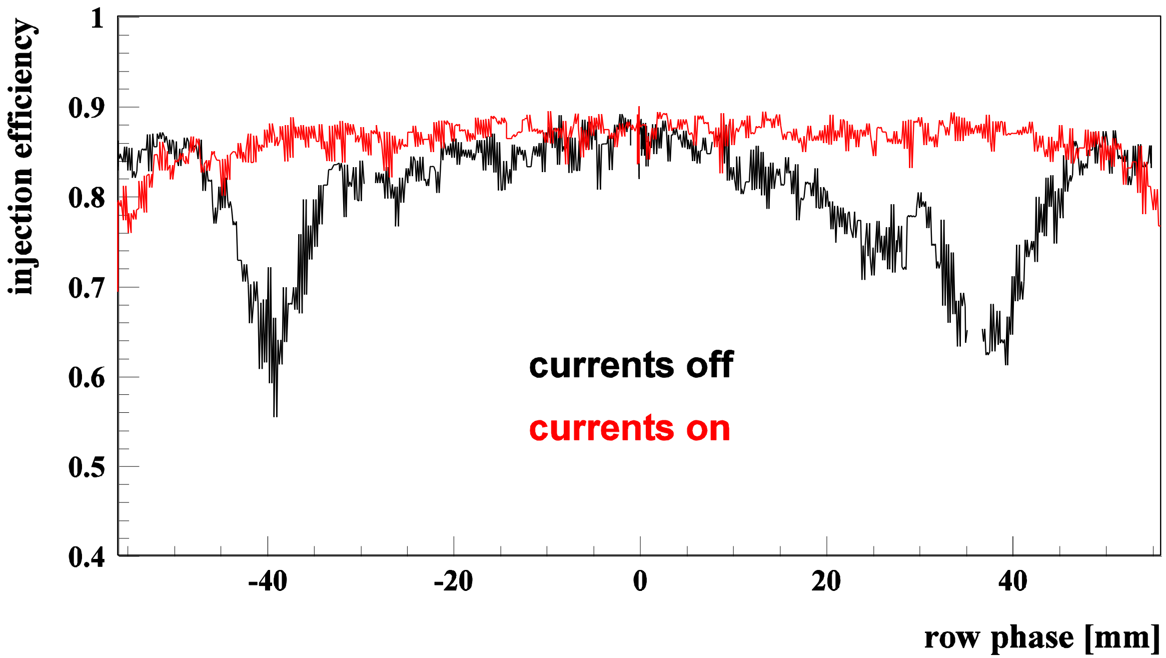


horizontal and vertical tunes vs horizontal displacement:
black: tune correction off, wires off
blue: tune correction on, wires off
red: no tune correction, wires on



source size variation with row phase of the UE112 at gap = 24mm in the elliptical mode (top) and the inclined mode (bottom). Black, blue: currents switched off; red, magenta: currents switched on.

UE112: Recovery of Injection Efficiency



Active shimming of the UE112 is essential for a top-up mode.
- inclined mode: no alternative to active compensation
- elliptical mode: improvement of passive shimming results

The next step: Compensation for universal mode

Static multipoles:

Complete description of two dimensional straight line field integral distributions on a source free circular disc

$\overline{F} = \overline{B}_x - i\overline{B}_y$ is an analytic function; the bar indicates a straight line integration

Cauchy Riemann relations:
$$\frac{\partial \overline{B}_x}{\partial x} = \frac{\partial(-\overline{B}_y)}{\partial y}$$
$$\frac{\partial(-\overline{B}_y)}{\partial x} = -\frac{\partial \overline{B}_x}{\partial y}$$
are equivalent to the 2D-Maxwell equations

"Dynamic multipoles" (DM):

$\widetilde{F} = \widetilde{B}_x - i\widetilde{B}_y$ is not an analytic function; the tilde indicates an integration along a wiggling trajectory

Cauchy Riemann relations are not fullfilled:
$$\vec{\nabla}\cdot(\widetilde{B}_x, \widetilde{B}_y) = \partial\widetilde{B}_x/\partial x + \partial\widetilde{B}_y/\partial y \propto -f_{002yx} + f_{002xy} = 0$$
$$\left(\vec{\nabla}\times(\widetilde{B}_x, \widetilde{B}_y)\right)_z = \partial\widetilde{B}_y/\partial x - \partial\widetilde{B}_x/\partial y \propto f_{002xx} + f_{002yy} \neq 0$$

Note: By principle "dynamic multipoles" can not be compensated with shims which are usually described by static field integrals

„Dynamic Multipoles“ of Planar Undulator

Static regular multipoles are usually described by:

$$\bar{B}_y^{(n)} = \sum_{i=1}^{\lfloor n/2 \rfloor} (-1)^{i-1} \bar{a}_{yni} \cdot x^{n+1-2i} y^{2(i-1)}$$

$$\bar{B}_x^{(n)} = \sum_{i=1}^{\lfloor n/2 \rfloor} (-1)^{i-1} \bar{a}_{xni} \cdot x^{n-2i} y^{2(i-1)+1},$$

In contrast the **“Dynamic multipoles”** of a planar undulator with only one transverse and one long. Fourier component:

$$\tilde{B}_y^{(n)} = -\delta_{0,\mathrm{mod}(n,2)} \frac{L}{8B\rho k^2} (-(\frac{k_x^3}{k_y^2} + k_x)(-1)^{n/2-1} \frac{1}{(n-1)!} (2k_x)^{n-1} x^{n-1} +$$
$$(\frac{k_x^3}{k_y^2} - k_x) \sum_{i=1}^{n/2} (-1)^{i+1} \frac{1}{(2i-1)!} \frac{1}{(n-2i)!} (2k_x)^{2i-1} (2k_y)^{n-2i} x^{2i-1} y^{n-2i}),$$

$$\tilde{B}_x^{(n)} = \delta_{0,\mathrm{mod}(n,2)} \frac{L}{8B\rho k^2} ($$
$$-\frac{k_x^2}{k_y^2} k_y \sum_{i=1}^{n/2} (-1)^{i+1} \frac{1}{(2i-2)!} (2k_x)^{2i-2} x^{2i-2} \frac{1}{(n-2i+1)!} (2k_y)^{n-2i+1} y^{n-2i+1} +$$
$$k_y \sum_{i=1}^{n/2} (-1)^{i+1} \frac{1}{(2i-2)!} (2k_x)^{2i-2} x^{2i-2} \frac{1}{(n+1-2i)!} (2k_y)^{n+1-2i} y^{n+1-2i} +$$
$$(\frac{k_x^2}{k_y^2} k_y + k_y) \frac{1}{(n-1)!} (2k_y)^{n-1} y^{n-1}).$$

„Dynamic Multipoles“ of Planar Undulator
Special Cases

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Case 1:
The undulator focusses horizontally with the same strength as vertically

$$\tilde{B}_y^{(n)} \propto -\sum_{i=1}^{n/2} \tilde{a}_{yni} \cdot x^{n+1-2i} y^{2(i-1)}$$

$$\tilde{B}_x^{(n)} \propto \sum_{i=1}^{n/2} \tilde{a}_{xni} \cdot x^{n-2i} y^{2(i-1)+1}.$$

Case 2:
The undulator defocusses horizontally
And the period length is long: $k_x^2 \approx k_y^2$

$$\tilde{B}_y^{(n)} \propto \tilde{b}_{yn} \cdot (-1)^{n/2-1} \cdot x^{n-1}$$

$$\tilde{B}_x^{(n)} \propto \tilde{b}_{xn} \cdot y^{n-1}.$$

Case 3:
The undulator has infinitely wide poles

$$\tilde{B}_y^{(n)} \propto 0$$

$$\tilde{B}_x^{(n)} \propto \tilde{c}_{xn} \cdot y^{n-1}.$$

Note: In all cases the “dynamic multipoles (DM)” are principally different from static multipoles

$$\bar{B}_y^{(n)} = \sum_{i=1}^{\lfloor n/2 \rfloor} (-1)^{i-1} \bar{a}_{yni} \cdot x^{n+1-2i} y^{2(i-1)}$$

$$\bar{B}_x^{(n)} = \sum_{i=1}^{\lfloor n/2 \rfloor} (-1)^{i-1} \bar{a}_{xni} \cdot x^{n-2i} y^{2(i-1)+1},$$

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Why does shimming of DM work at all?

Shimming of DM in the midplane has no principle limitations, but vertical off-axis effects are enhanced; this is acceptable because:

- usually, vertical beta-function smaller than horizontal beta functions
- usually vertical emittance smaller than horizontal emittance
- large particle amplitudes occur during horizontal injection

What about gap dependency?

DM are expected to drop off much faster than shim field integrals due B^2 dependency, but:

- detailed considerations show similar gap dependence of dynamic multipoles and static multipoles for long period lengths
- DM scale with square of period length; Murphy’s Law does not apply ☺

Summary and Outlook

Fast analytic, symplectic GF-based tracking scheme
one step per undulator is possible

Analytic description of undulator fields and shim field integrals
simplifies interface

Analytic expressions for analytic kicks

Extension of method to other undulator structures
and accelerator magnets