Coaxial Elliptic Helical Undulator

A design concept for an undulator that generates polarized magnetic fields of linear, circular, and elliptical modes

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Outline

- Motivated from "Is it possible to change the nature of the circular polarization and to obtain linearly polarized radiation from the same undulator?" [1]

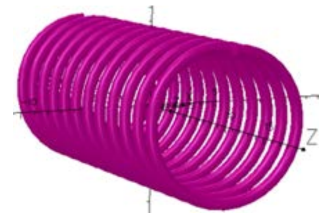
- Magnetic field of a solenoid
- Characteristics of helical undulator magnetic field
- Coaxial (circular) helical undulator
- Coaxial elliptic helical undulator
- Compare the polarized fields with those of an APPLE-II
- Conclusion

[1] D.F. Alferov *et al*., Sov. Phys. Tech. Phys. 21 (1976) 1408

Magnetic field of an infinitely long helical solenoid

- One-layer solenoid is always helical with a winding pitch angle
- The helix is wound on radius r_0 with a filamentary wire
- Calculate on-axis transverse fields from the Biot-Savart law [2]

$$B_{axis}^{z} = \frac{\mu_0 I_1}{4\pi}\int \frac{[d\mathbf{s}\times\mathbf{r}]_z}{r^3} = \mu_0 n_s I_1$$



n_s = # of turns per unit length

$$B_{axis}^{y} = \frac{\mu_0 I_1}{4\pi}\int \frac{[d\mathbf{s}\times\mathbf{r}]_y}{r^3}$$

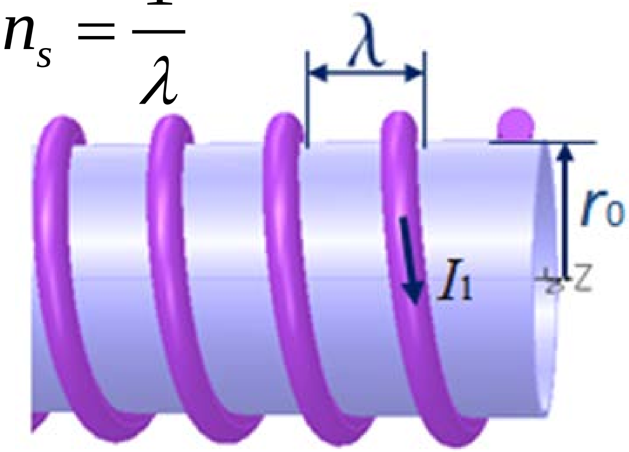
$$n_s = \frac{1}{\lambda}$$

$$= \frac{\mu_0 I_1}{\lambda}\left\{\left(\frac{2\pi}{\lambda}r_0\right)K_0\left(\frac{2\pi}{\lambda}r_0\right) + K_1\left(\frac{2\pi}{\lambda}r_0\right)\right\}$$

$$\sim \exp\left(-2\pi\frac{r_0}{\lambda}\right)$$

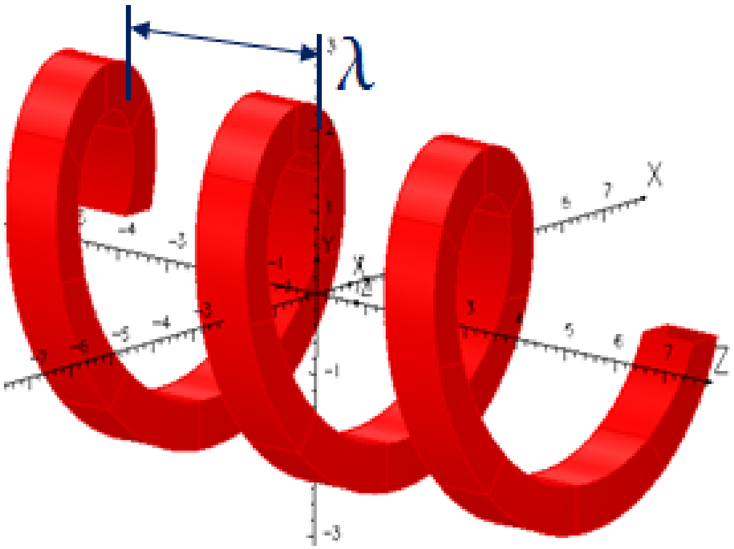
$$B_{axis}^{x} = 0$$

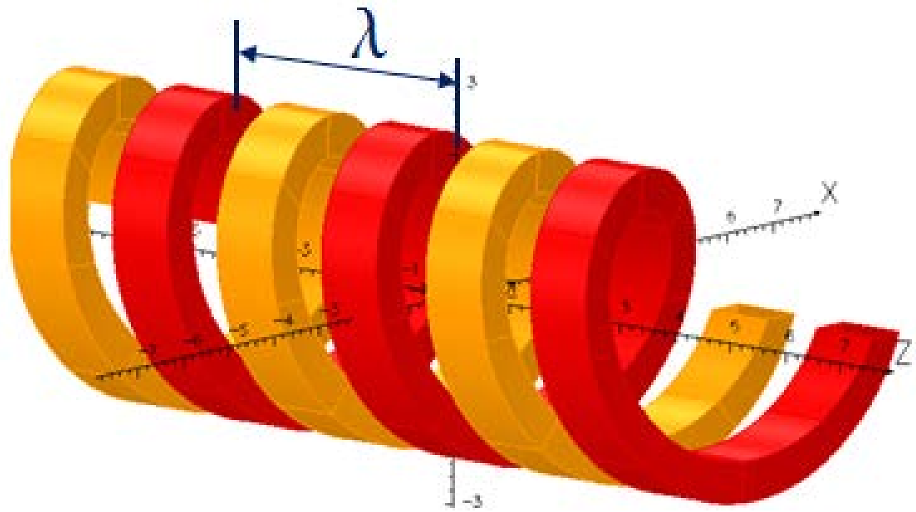
$$\mathbf{B}_{axis}^{tran} = B_{axis}^{tran}\left\{\hat{x}\cos\left(\frac{2\pi}{\lambda}z\right) + \hat{y}\sin\left(\frac{2\pi}{\lambda}z\right)\right\}$$



[2] W.R. Smyth, *Static and Dynamic Electricity* (McGraw-Hill, New York, 1939), p. 272

Rectangular conductors

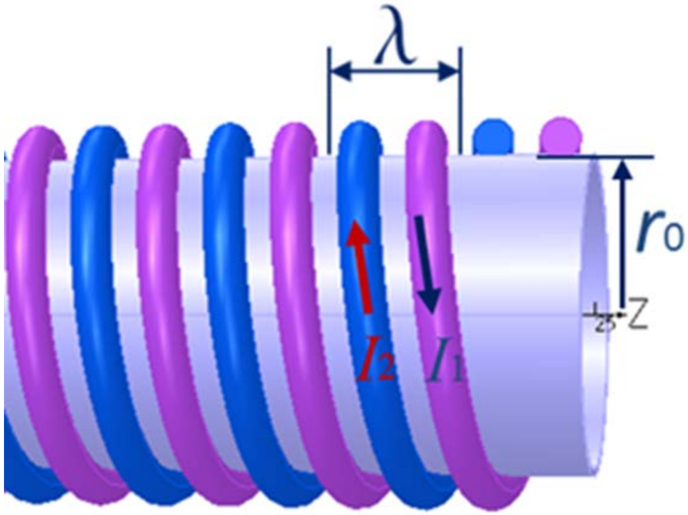




B_x in the previous slide at z = 0 is zero because the x-axis goes through the both red and yellow conductors and the field integral of the Biot-Savart law is anti-symmetrical with respect to the coil-winding pitch angle.

Bifilar Helix as a Helical Undulator

$$\mathbf{B}_{axis}^{tran} = B_{axis}^{tran}\{\hat{r}\cos(\frac{2\pi}{\lambda}z-\phi)+\hat{\phi}\sin(\frac{2\pi}{\lambda}z-\phi)\}$$



$$\mathbf{B}_{axis}^{tran} = B_{axis}^{tran}\{\hat{x}\cos(\frac{2\pi}{\lambda}z)+\hat{y}\sin(\frac{2\pi}{\lambda}z)\}$$

$$B_{axis}^{z} = \frac{\mu_0}{\lambda}(I_1-I_2) \rightarrow 0$$

$$\rightarrow 2I$$

$$B_{axis}^{tran} = \frac{\mu_0(I_1+I_2)}{\lambda}\left\{(\frac{2\pi}{\lambda}r_0)K_0(\frac{2\pi}{\lambda}r_0)+K_1(\frac{2\pi}{\lambda}r_0)\right\}$$

- Assumes infinitesimal cross section of the wire
- The on-axis field is proportional to the current in the wire

[3] B.M. Kincaid, J. Appl. Phys. 48 (1977) 2684
[4] J.P. Blewett and R. Chasman, J. Appl. Phys. 48 (1977) 2692

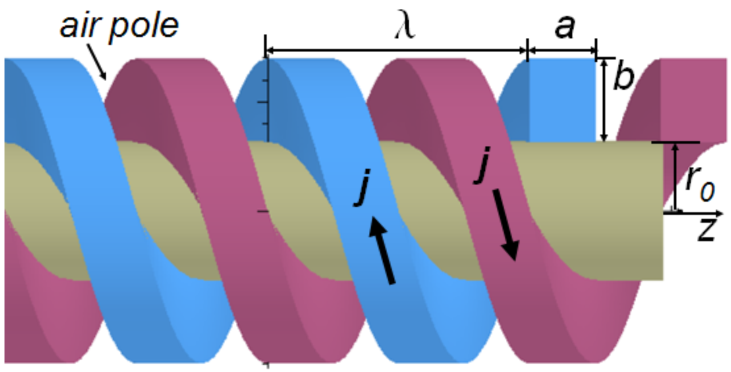
(Circular) Helical Undulator

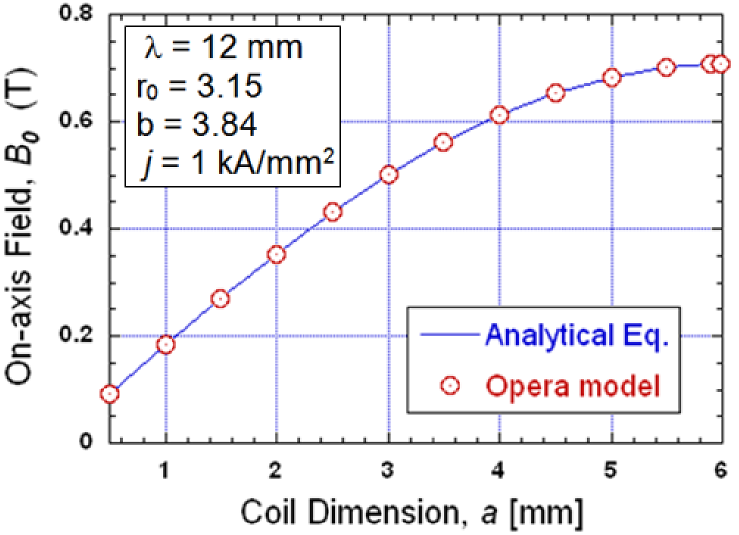
$$B_{axis}^{n=1} = \frac{\mu_0 2 j \lambda}{\pi} \sin(k\frac{a}{2}) \int_{r_0}^{r_0+b} \{krK_0(kr) + K_1(kr)\} \frac{dr}{\lambda}$$

$$(k = \frac{2\pi}{\lambda})$$

j: coil current density

$$\mathbf{B}_{axis}^{tran} = B_{axis}^{n=1} \{\hat{x}\cos(kz) + \hat{y}\sin(kz)\}$$





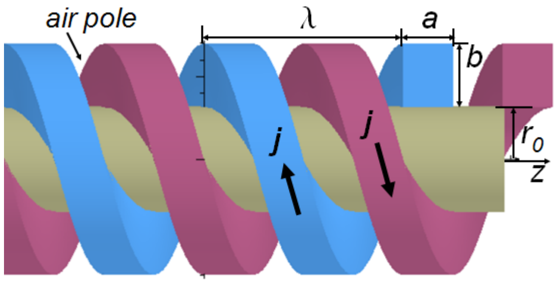
- We have an analytical expression with coil cross sections [4]
- Agrees with model calculations: field within 3×10^{-5}, higher harmonics << 2×10^{-7}
- When undulator dimensions are scaled according to λ, the field remains unchanged for $j\lambda$ = constant

[5] S.H. Kim, Nucl. Instr. and Meth. A 584 (2008) 266

(Circular) Helical Undulator: off-axis field

$$B^n_{axis} = \frac{2\mu_0 j\lambda}{\pi}\sin\left(\frac{nka}{2}\right)\int_{r_0}^{r_0+b}\{nkrK_{n-1}(nkr)+K_n(nkr)\}\frac{dr}{\lambda} \qquad (k=\frac{2\pi}{\lambda})$$

$$\mathbf{B}(0\le r<r_0) = \sum_{n=1,3,5..}^{\infty} B^n_{axis}\cdot\{\hat{r}B^n_r + \hat{\phi}B^n_\phi + \hat{z}B^n_z\}$$

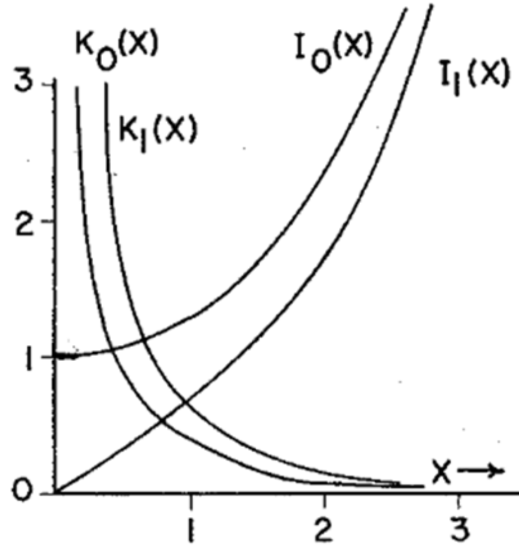


$$B^n_r = [I_{n-1}(nkr)+I_{n+1}(nkr)]\cdot\cos[n(kz-\phi)]$$

$$B^n_\phi = (\frac{2}{kr})I_n(nkr)\cdot\sin[n(kz-\phi)]$$

$$B^n_z = (-2)I_n(nkr)\cdot\sin[n(kz-\phi)]$$

I_n, K_n: modified Bessel functions



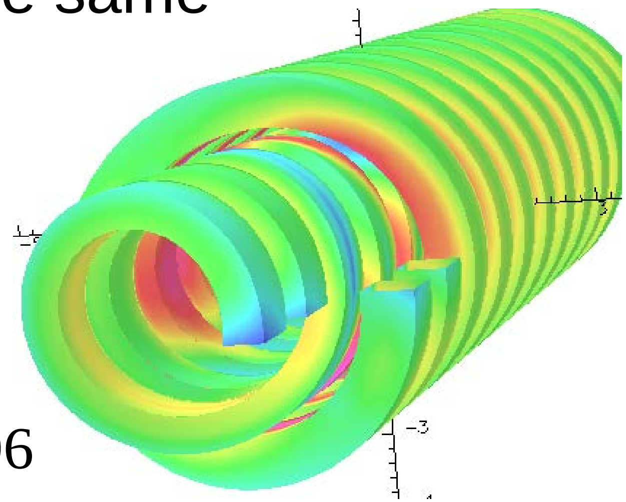
$$B^1_r = [1+\frac{3(kr)^2}{8}+\frac{5(kr)^4}{192}+\ldots]\cos(kz-\phi)$$

$$B^1_\phi = [1+\frac{(kr)^2}{8}+\frac{(kr)^4}{192}+\ldots]\sin(kz-\phi)$$

$$B^1_z = -[kr+\frac{(kr)^3}{8}+\ldots]\sin(kz-\phi)$$

Coaxial (Circular) Helical Undulator

- Inner/outer two helical undulators have coil-winding pitch angles in the opposite directions along the same undulator axis

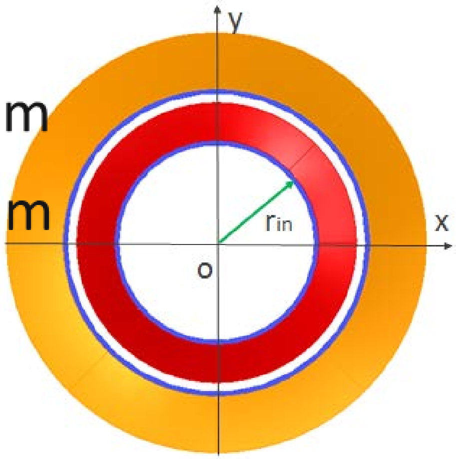


$$\begin{cases} \mathbf{B}_{in} = B_{axis}^{in}\{\hat{r}\cos(kz-\phi)+\hat{\phi}\sin(kz-\phi)\} \\ \mathbf{B}_{out} = B_{axis}^{out}\{\hat{r}\cos(kz+\phi)-\hat{\phi}\sin(kz+\phi)\} \end{cases}$$

$$\begin{cases} \mathbf{B}_{in} = B_{axis}^{in}\{\hat{x}\cos(kz)+\hat{y}\sin(kz)\} \\ \mathbf{B}_{out} = B_{axis}^{out}\{\hat{x}\cos(kz)-\hat{y}\sin(kz)\} \end{cases} \qquad \begin{cases} B_{axis}^{in}(T) = 0.8696 \\ B_{axis}^{out}(T) = 0.4881 \end{cases}$$

Period = 38 mm

r_{in} = 11 mm



- Calculated *~j(critical) @4.2 K*
- Linear polarizations, for example:

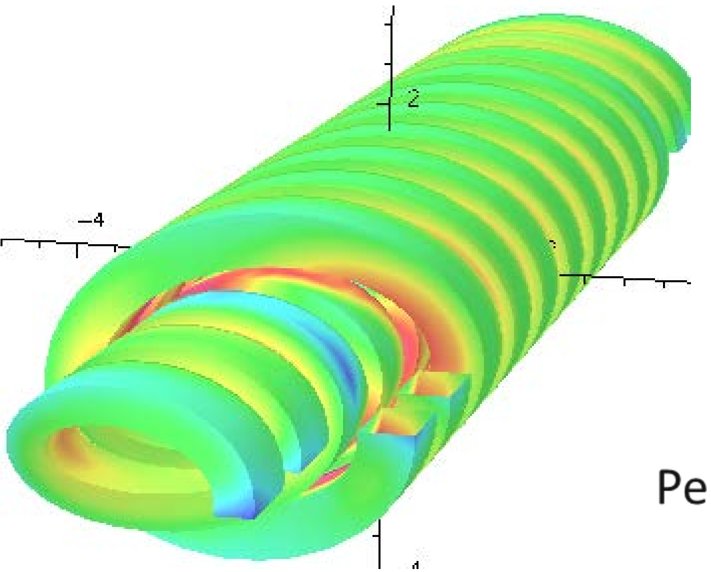
$$I_{in} \rightarrow \frac{B_{axis}^{out}}{B_{axis}^{in}} I_{in} \qquad I_{out} \rightarrow \pm I_{out}$$

Coaxial Elliptic Helical Undulator

- Could not derive an analytical expression yet for an ellipse
- By modifying the cross section from the circular to an ellipse,

  Bx → ~1.2 Bx   By → ~2 By

$$\begin{cases} \mathbf{B}_{in} = B_{axis}^{in}\{\hat{x}f_{in}\cos(kz) + \hat{y}\sin(kz)\} \\ \mathbf{B}_{out} = B_{axis}^{out}\{\hat{x}f_{out}\cos(kz) - \hat{y}\sin(kz)\} \end{cases}$$

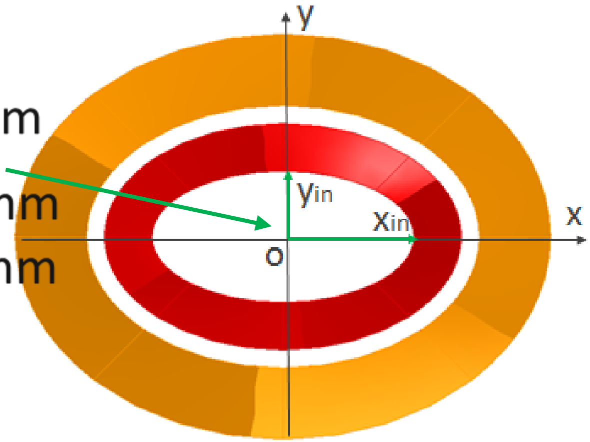


$$\begin{cases} \mathbf{B}_{in}(T) = 1.8175\cdot\{\hat{x}\cdot 0.6335\cdot\cos(kz) + \hat{y}\sin(kz)\} \\ \mathbf{B}_{out}(T) = 1.0048\cdot\{\hat{x}\cdot 0.6216\cdot\cos(kz) - \hat{y}\sin(kz)\} \end{cases}$$

Period = 38.0 mm

x_{in} = 11.0 mm

y_{in} = 5.5 mm



Calculated polarized fields are compared with those of an APPLE-II [6]

Type	Period/gap (mm)	Circular field (T)	Vertical field (T)	Horizontal field (T)	Elliptical field (T)
APPLE-II*	38/9.5	0.566	0.9292	0.7139	B_y = 0.8486 B_x = 0.2908
Elliptic Helical	38/11	1.407	1.990	1.261	B_y B_x 1.421 0.642 0.422 2.258 1.152 0.818 0.625 -1.005
Elliptic Helical	26.6/11	0.580	1.305	0.696	B_y B_x 0.974 0.533 0.282 1.881 0.628 1.207 0.346 -0.674

*Calculated by S. Sasaki with B_r = 1. 27 T

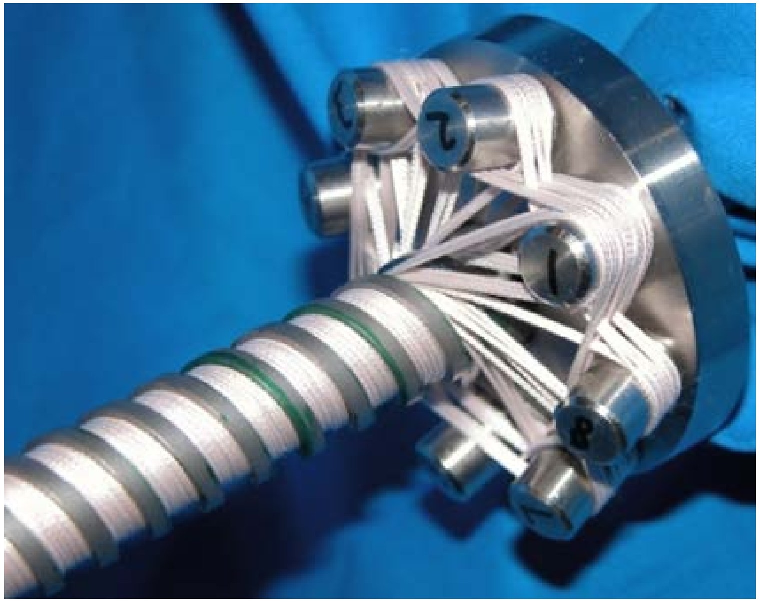
[6] S. Sasaki, Nucl. Instr. And Meth. A 347 (1994) 83

$j\,\lambda$ constant scaling law for an Elliptic Helical Undulator

- U38: λ = 38 mm, j = 1.0 kA/mm^2
- U76: λ = 76 mm, j = 0.5 kA/mm^2
- U38x0.7: λ = 26.6 mm, j = 1/0.7 kA/mm^2
- → $j\,\lambda$ = 38 kA/mm for the three
- Have the same calculated on-axis fields within ~1 mT

Issues

- Tolerances for the coaxial alignment
- Effective magnetic lengths of the inner and outer units
- Off-axis field
- One end for the inner unit
- - - -



Conclusion

- Proposed a coaxial elliptic helical undulator for use in a storage ring
- The undulator partially follows the $j\lambda$ constant scaling law
- Calculated polarized fields for a period of 38 mm were about twice of those for an APPLE-II
  - Fields for a period of 26.6 mm were slightly higher than those for the 38-mm APPLE-II
- Need further analysis