Beam Dynamics of APS-U beam with the Higher-Harmonic Cavity

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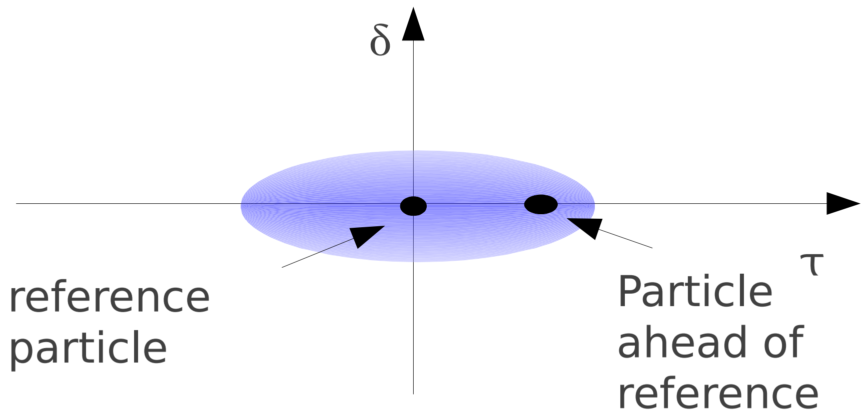
Outline

- Higher-Harmonic Cavity (HHC) = Bunch lengthening cavity = Landau cavity
  - Reasons for Higher-Harmonic Cavity
- Longitudinal Motion Review
  - Coordinates, equations of motion
  - Potential well, Hamiltonian
- Expectations from HHC
- Choices of parameters for HHC
- Why rely on tracking

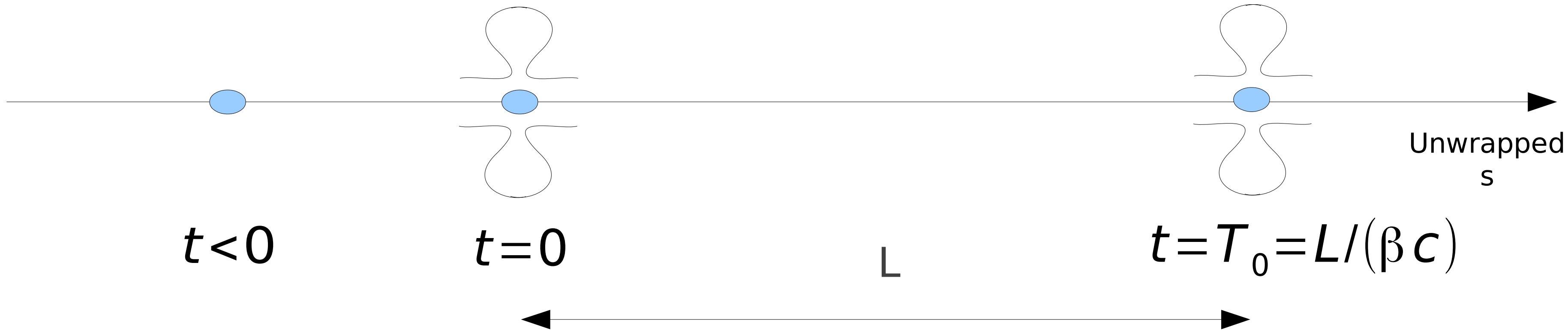
Longitudinal Coordinates in Bunches

Time coordinate is τ (position coordinate is $z=c\tau$)
Momentum coordinate is $\delta=(p-p_0)/p_0$



What an rf Cavity Does

- Assume one rf cavity (main rf system) in ring with e.m. fields timed to give the correct energy to a particle at every turn
- At time t = 0 bunch is centered in cavity, and gets some energy gain (or loss) from the electric fields present at the time

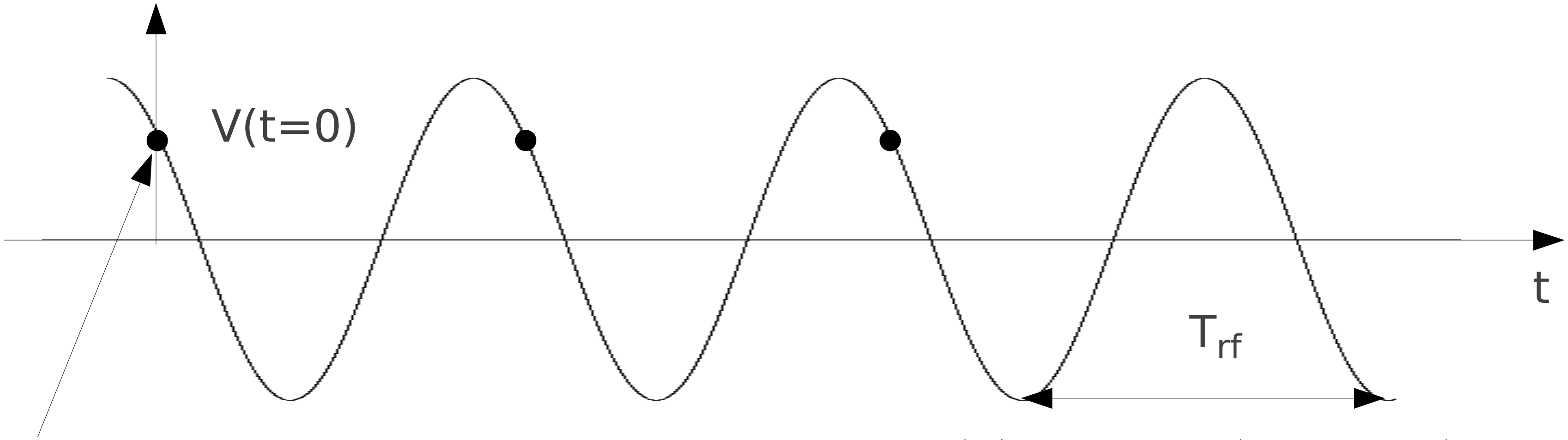


- Then at time t = L/v = L/(βc) the bunch returns to the cavity and gets another step in energy. This repeats every turn.

RF Cavity Voltage

Net energy gain in terms of cavity voltage: $\Delta E(t) = eV(t)$

The voltage is an integral of E_z within the cavity gap, which varies with changes in arrival time



Bunches passes once through cavity at t=0 very quickly

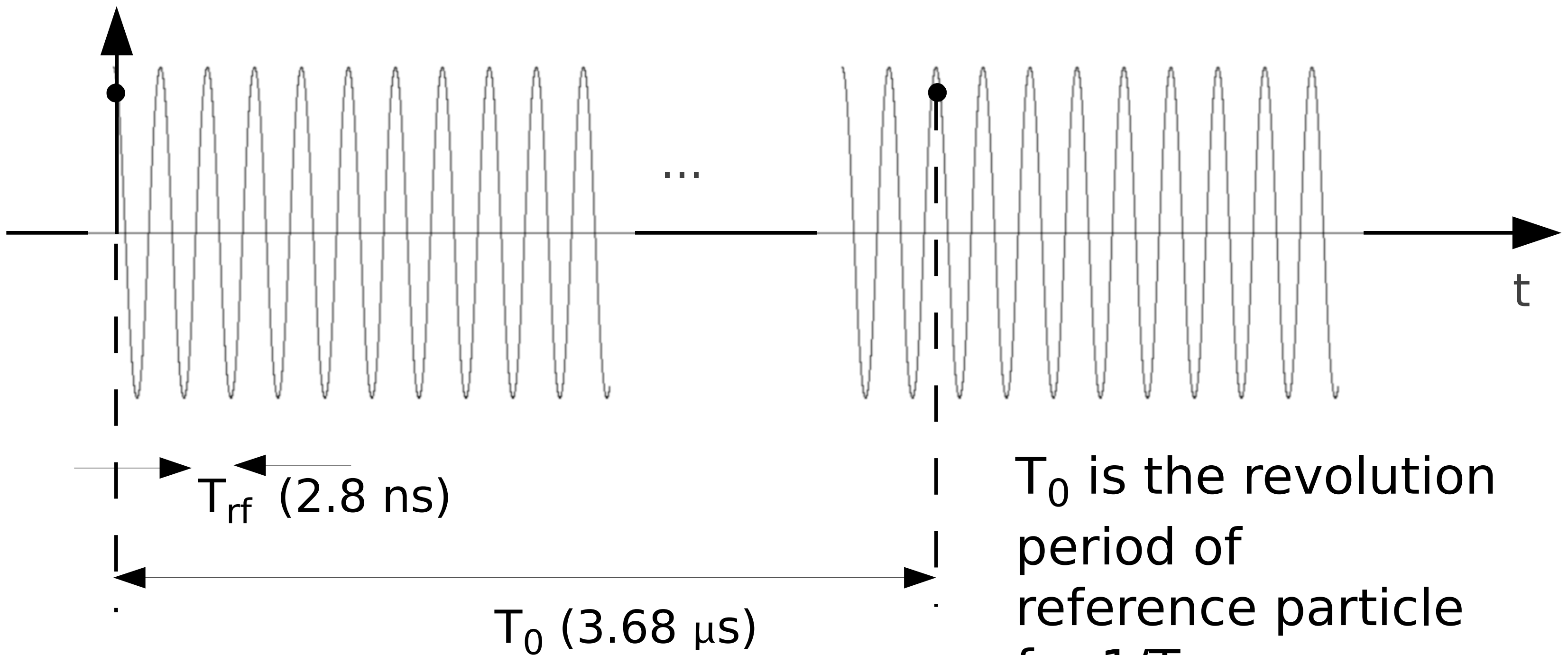
$$V(t) = V_0 \cos(\omega_{rf} t + \varphi_s)$$

$$\omega_{rf} = 2\pi f_{rf}$$

$$f_{rf} = \frac{1}{T_{rf}}$$

$$U_0 = V_0 \cos(\varphi_s)$$

RF Cavity Voltage



T_0 is the revolution period of reference particle $f_0=1/T_0$

rf phase oscillations before the same particle would return

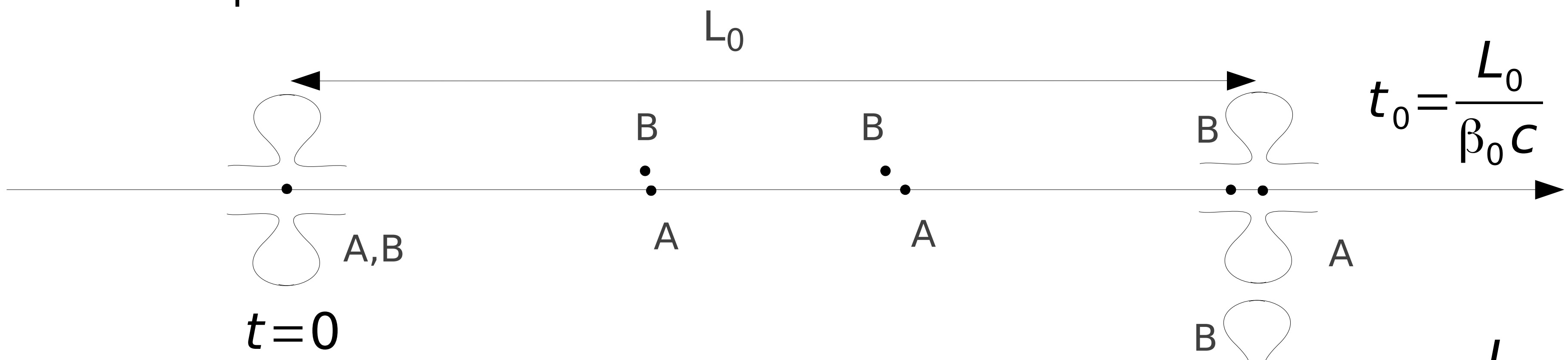
$$f_{rf}=\frac{1}{T_{rf}}=h\left(\frac{\beta c}{L}\right)=\frac{h}{T_0}=hf_0$$

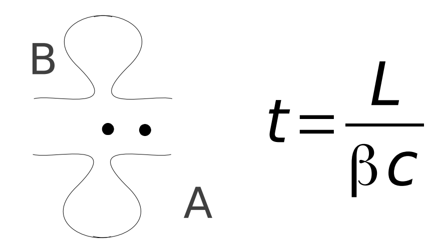
h is the harmonic number 1296 of the rf frequency f_{rf}/f_0

Off-momentum Particle

A particle with momentum error ($\delta=(p-p_0)/p_0$)will not stay in phase with the rf fields because of path length change

Take two particles (A: $\delta=0$ and B: $\delta>0$) that arrive at t=0 on the first pass.





Particle B arrives at different time at the cavity

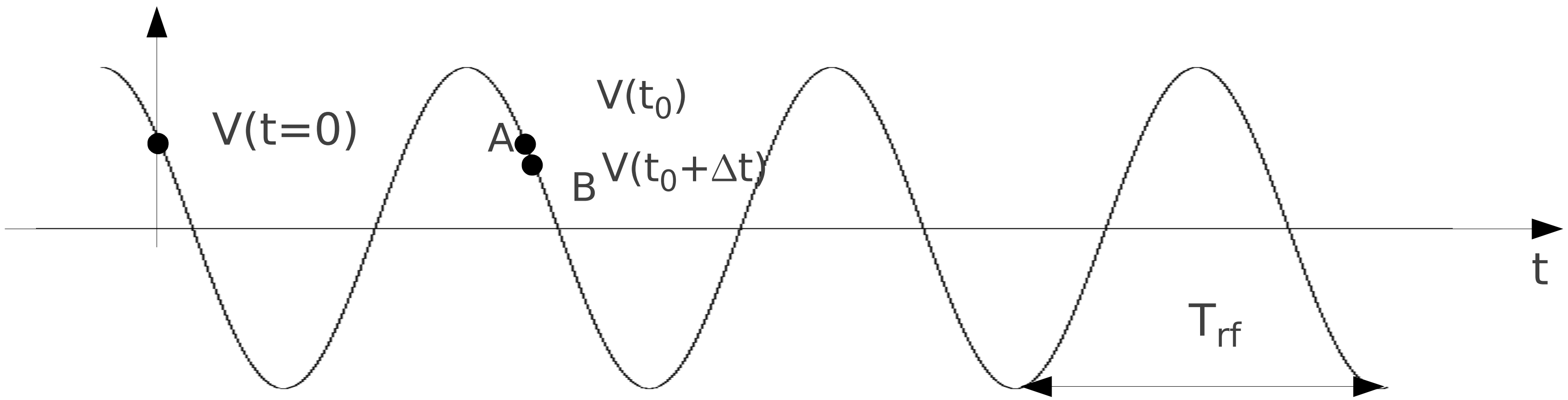
Particle B won't see the same field and net energy gain as A

Off-Momentum Particle

The relative difference in time is $$\frac{\Delta t}{t}=\left(\alpha_p-\frac{1}{\gamma^2}\right)\delta=\eta_c\delta$$

where $$\alpha_p=\frac{\Delta L/\delta}{L_0}=\frac{1}{L_0}\int_0^{L_0} ds\,h\,\eta(s)=5.66\times10^{-5}$$

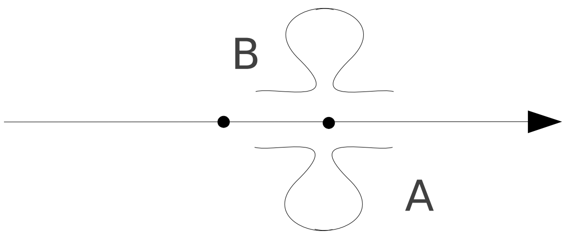
For δ>0, Δt will be positive



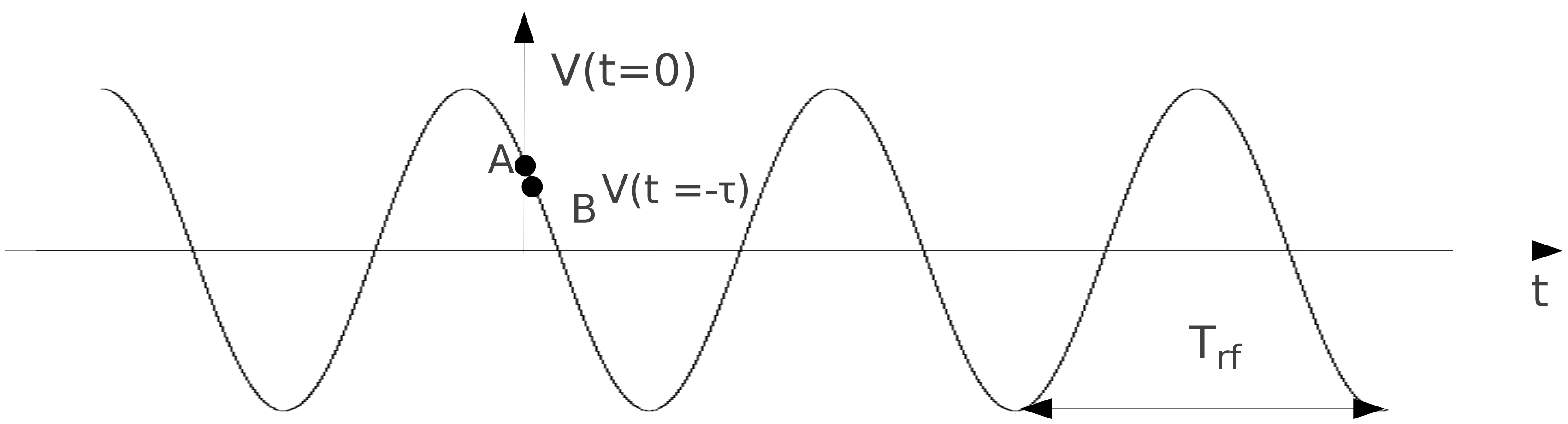
$V_B<V_A$ because of the slope of the voltage

Mis-timed Particle

Time coordinate for B is $\tau < 0$. For A, $\tau = 0$.



Time coordinate acts as a longitudinal distance coordinate, $z = c\tau$



Mis-timed Particle

Suppose that $V(t)=V_0\cos\left(\omega_{rf}t+\varphi_s\right)$ with $U_0=V_0\cos(\varphi_s)$

Then particle with coordinate τ gets momentum change (Δδ) relative from reference particle using

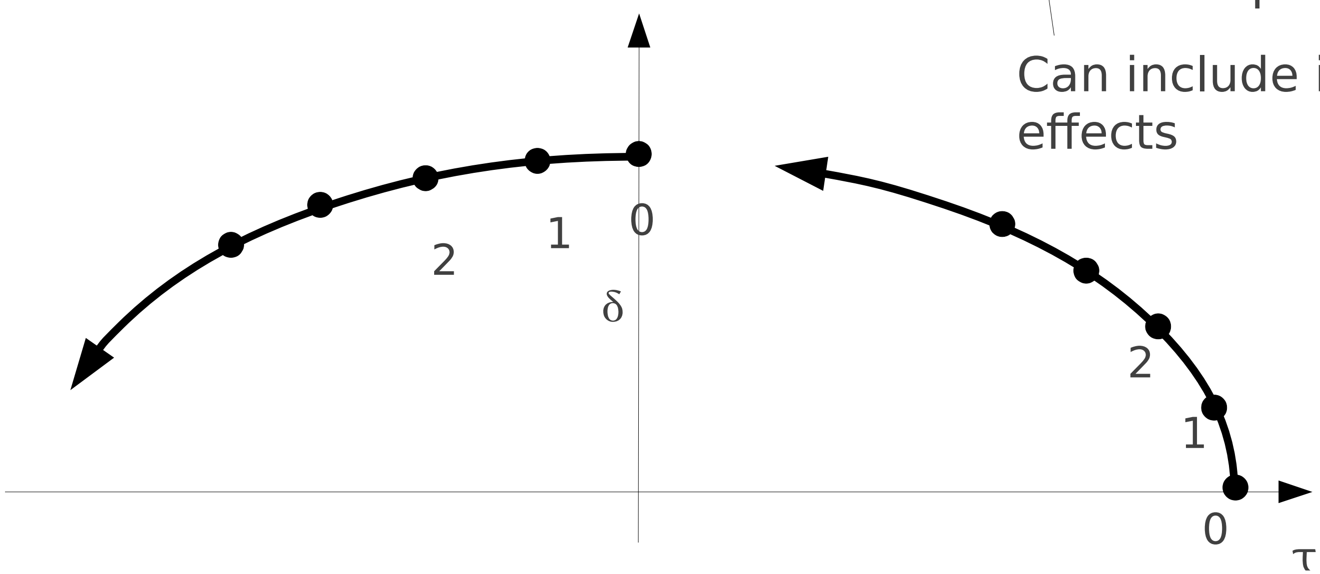
$$\Delta V=V(t+\tau)-V(t)=-\tau\omega_{rf}V_0\sin(\varphi_s)$$

Two Coupled Difference Equations produce an Elliptical Motion

$$
\begin{aligned}
\frac{\tau_n - \tau_{n-1}}{T_0} &= -\alpha_c \delta \\
\delta_n - \delta_{n-1} &= \frac{1}{E}\left(eV_{\text{ext}}(\tau) + eV_{\text{ind}}(\tau) - U\right)
\end{aligned}
$$

Energy loss per turn

Can include impedance effects



Equations of Motion Turned into Hamiltonian

$$
\begin{aligned}
\dot{\tau} &= -\alpha_c \delta \\
\dot{\delta} &= \frac{1}{T_0 E}\left(eV_{\text{ext}}(\tau) + eV_{\text{ind}}(\tau) - U\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial H}{\partial \delta} &= \alpha_c \delta \\
\frac{\partial H}{\partial \tau} &= \frac{1}{T_0 E}\left(eV_{\text{ext}}(\tau) + eV_{\text{ind}}(\tau) - U\right)
\end{aligned}
$$

$$
\frac{H(\delta,\tau)}{H_0} = \frac{1}{2}\frac{\delta^2}{\sigma_\delta^2} + \underbrace{\frac{1}{T_0 E \alpha_c \sigma_\delta^2}\int_0^\tau \left(eV_{\text{ext}}(\tau) + eV_{\text{ind}}(\tau) - U\right) d\tau}_{\text{Potential}}
$$

Hamiltonian to Distribution

From reference below,

$$\frac{d^2 N}{d\delta\, d\tau} = \exp\left(-H(\delta,\tau)/H_0\right)$$

$$I(\tau) = C \exp\left(-\frac{1}{T_0 E \alpha_c \sigma_\delta^2} \int_0^\tau \left(eV_{\text{ext}}(\tau) + eV_{\text{ind}}(\tau) - U\right)\, d\tau\right)$$

This is the Haissinski equation for potential well distortion.

If there is no impedance the just plug in any arbitrary external voltage, then get current profile immediately

J. Haïssinski. Exact longitudinal equilibrium distribution of stored electrons in the presence of self-fields, Il Nuovo Cimento, 18(1), 72–82, 1973

Some Basic Parameters for Current Profile Formula

- Main RF: 4.1 MVpk
- Beam Energy: 6 GeV
- U_loss = 2.27 MeV/turn
- momentum compaction: 5.66e-5
- momentum spread: 0.096%

To Include Wakes

Potential can be integrated numerically, though iterations of the exponential expression are required

$$\int_{-\infty}^{\tau} V_{\text{ind}}(\tau)\, d\tau = \int_{-\infty}^{\tau} \int_{-\infty}^{\infty} I(\tau')W_\delta(\tau' - \tau)\, d\tau'$$

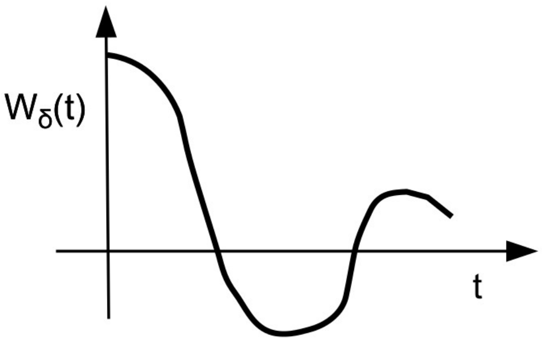


Figure 1: Schematic wake function as a function of elapsed time

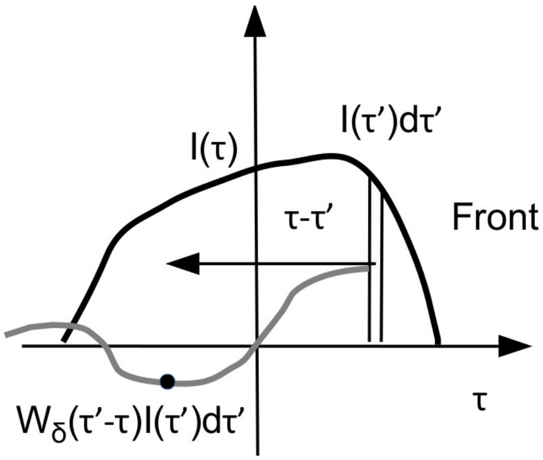


Figure 2: Schematic current profile with wake potential at τ due to a slice at τ'. The negative of $W_\delta(\tau' - \tau)I(\tau')\, d\tau'$ is the actual wake potential contribution.

Potential of Single RF and Double RF

- A single RF system will produce a linear restoring force and a quadratic potential, i.e. the usual Gaussian distribution

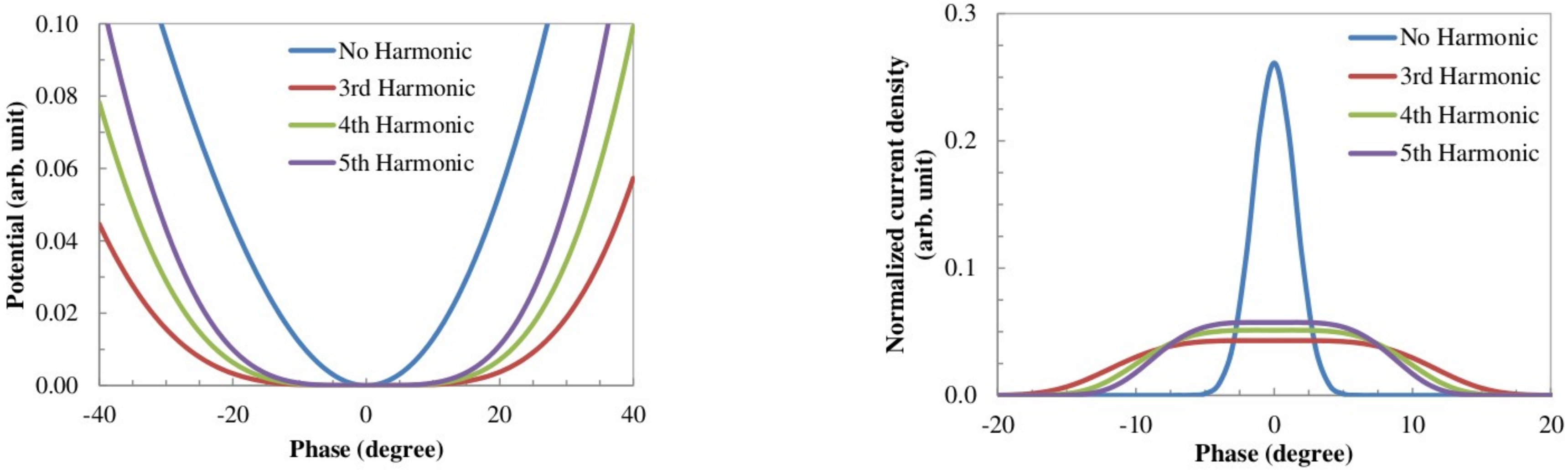
$$V(\tau) \sim \tau \qquad\qquad I(\tau) \sim \exp\left(-\tau^2/\sigma_\tau^2\right)$$

- A double RF system can be made to produce a quartic potential,

$$V(\tau) \sim \tau^3 \qquad\qquad I(\tau) \sim \exp\left(-A\tau^4\right)$$

Note that the momentum distribution remains Gaussian

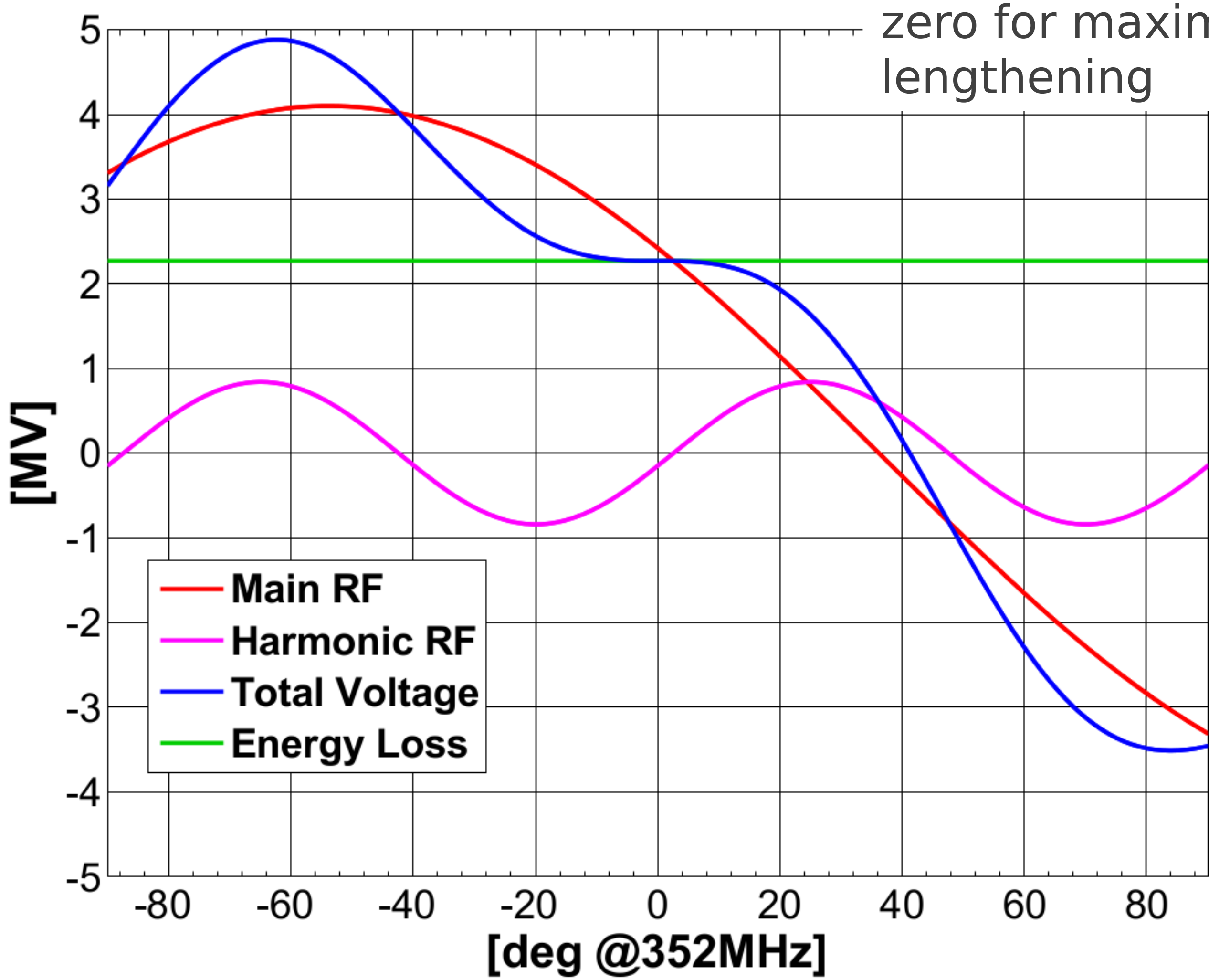
Several Choices of Harmonics



From CDR: The 4th harmonic is chosen as the baseline design based on a compromise between the calculated increase in the Touschek lifetime and the power-handling requirement for the rf coupler

External Voltage with HHC

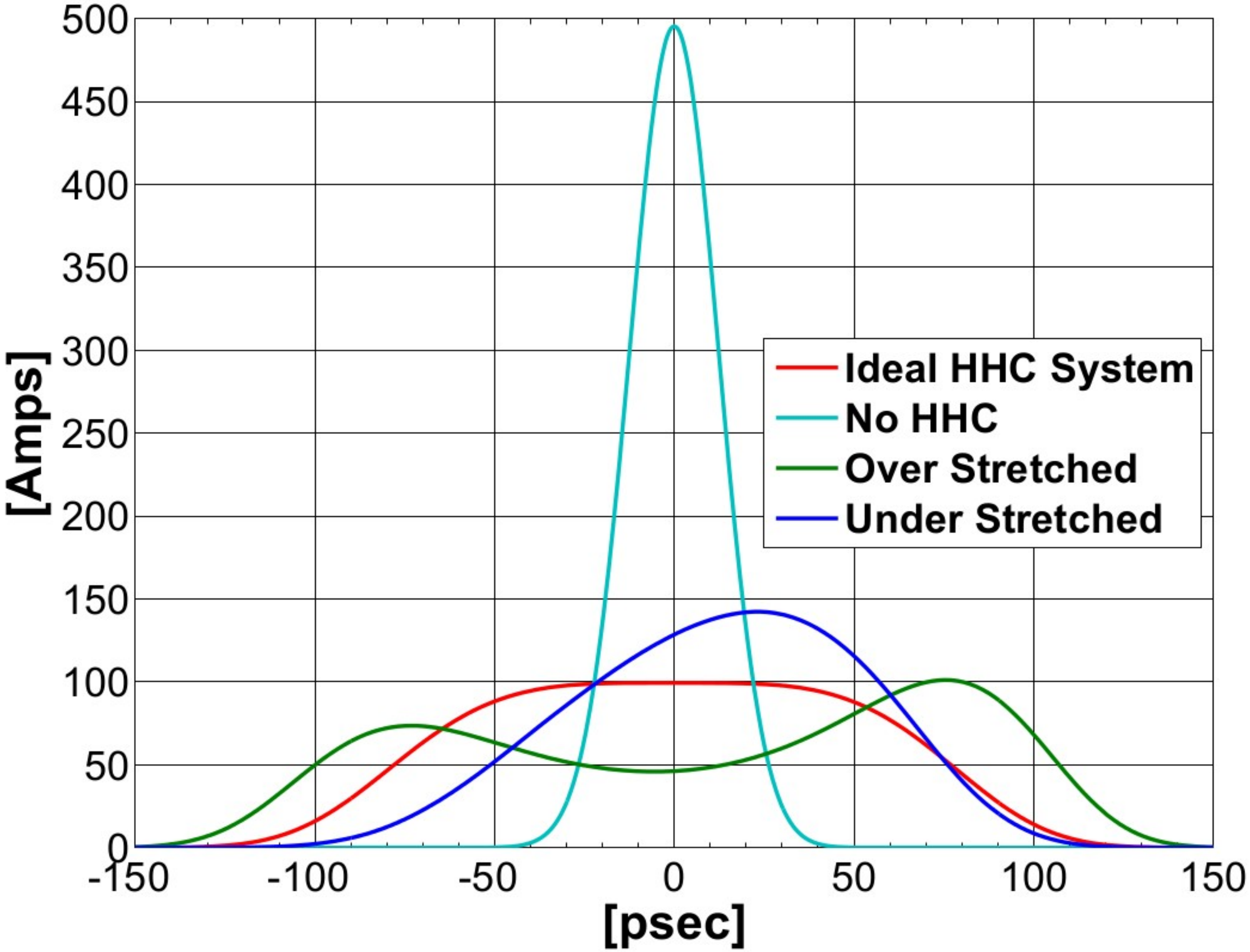
1st and 2nd derivative of V set to zero for maximum bunch lengthening



V_0 = 4.1 MV,
φ_0= 143.8 deg,
N=4,
kV_n = 0.84 MV,
and
φ_n = -10.36 deg

T. Berenc

Profile Calculated from Hamiltonian with HHC



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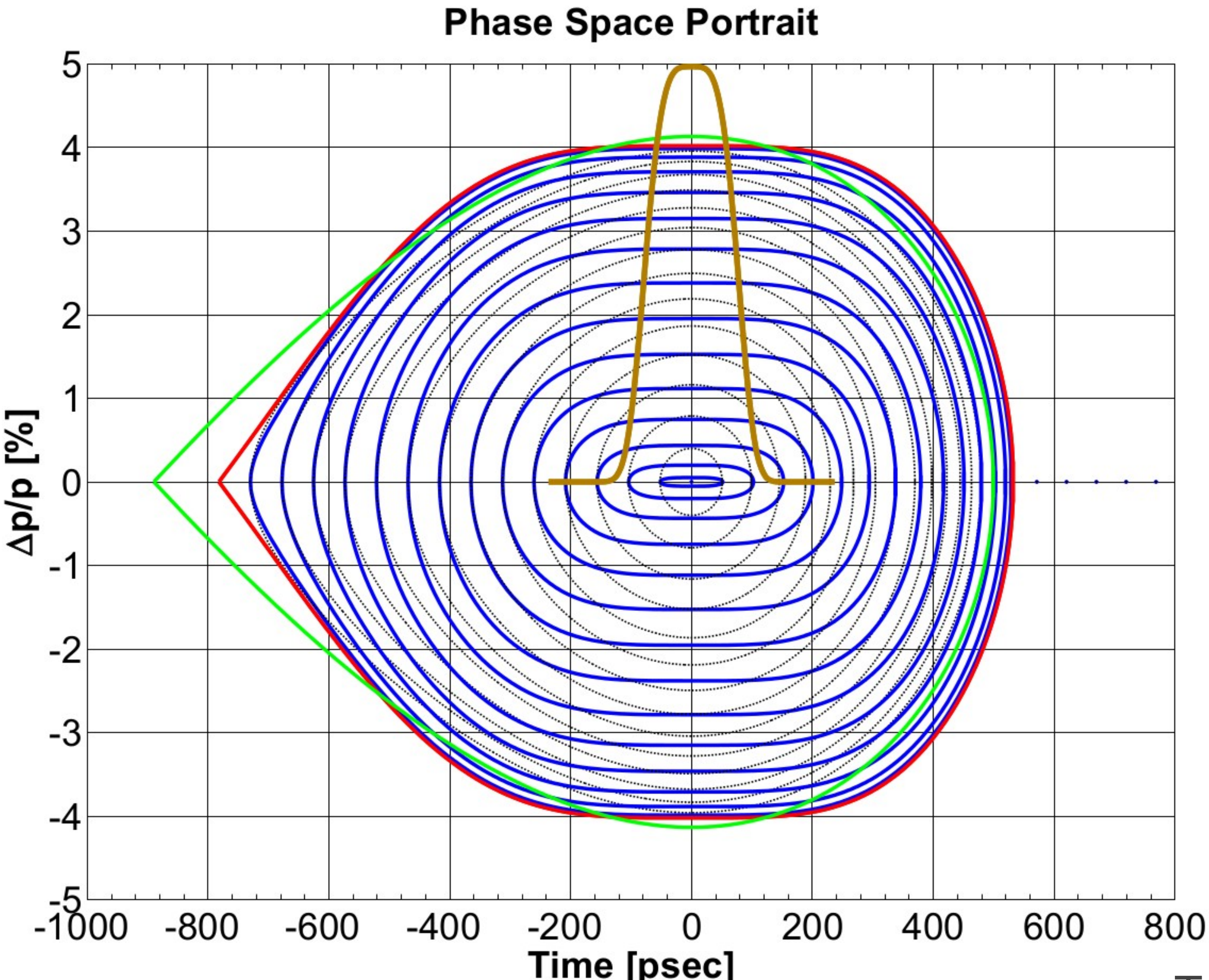
Trajectories without and with HHC



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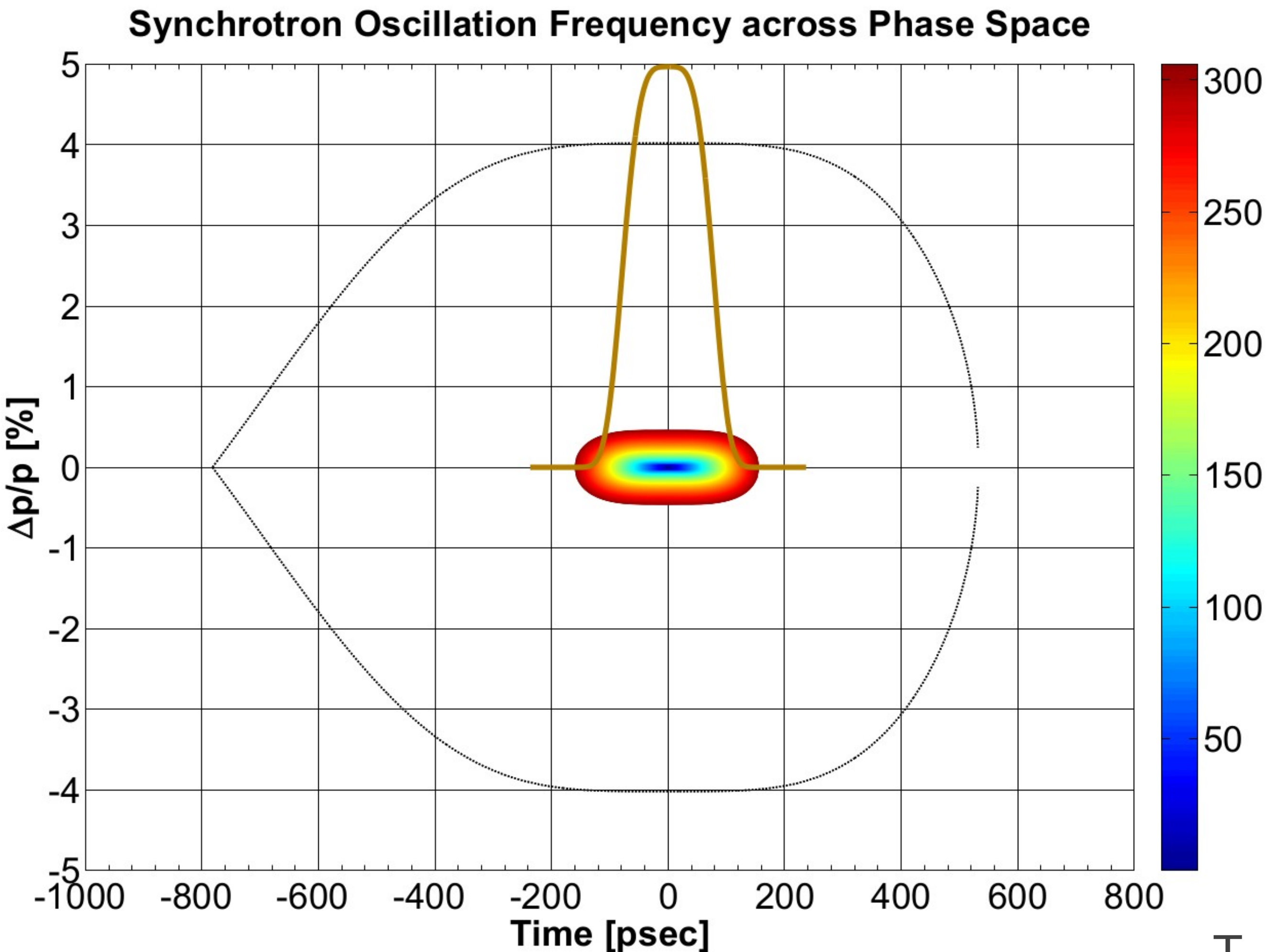
Frequency Dependence on Amplitude

Without HHC,
F_s = 700 Hz



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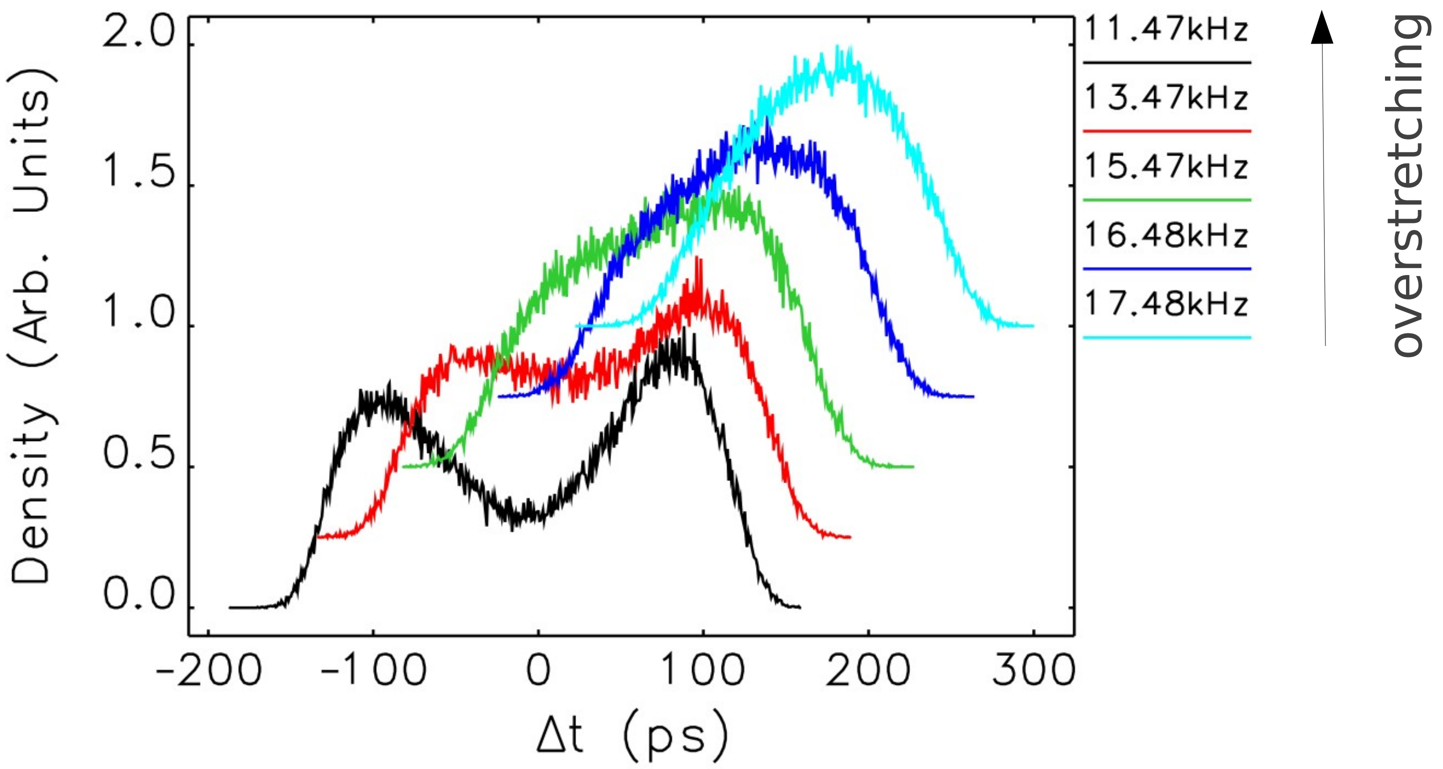
Passive mode of HHC

- Stored beam itself generates voltage in HHC that can serve to lengthen the bunch. Controlled by
  - beam current,
  - the cavity loaded Q_L,
    - High Q means high voltage
  - the cavity detuning ($\Delta f = f_h - nf_0$, where f_h is the HHC cavity resonant frequency
    - Lower Δf means higher voltage
- Above Q_L and Δf must be set to provide kV_n = 0.84 MV, and φ_n = -10.36 deg at 200 mA.
  - $Q_L = 2.3\times10^5$, Δf = 16.3 kHz
- For "overstretched" case, which is used as baseline
  - $Q_L = 6\times10^5$, Δf = 14 kHz and
- Active mode refers to external voltage applied, which is more complicated and costly

Overstretching as Seen in Tracking



Borland, CDR

Reduction of derivatives (Berenc)

- A double RF system zeros the 1st and 2nd derivatives
- A triple RF system zeros the first 4 derivatives
- Obviously, you can keep adding harmonics to zero an additional 2 derivatives each. For proton machines, ultimately you want a square wave; e.g. a barrier bucket system.

Limitation of Hamiltonian Expression

- Though it can handle Potential Well distortion and double RF, it doesn't handle some impedance issues like turbulent bunch lengthening
- Tracking studies can include all the impedance effects and non-linear effects
- Tracking studies permit studies of bunch population variation, missing bunches, injection from zero, beam loss from non-perfect matching of booster phase space, multi-bunch instabilities
  - See Borland's recent internal notes on most of these

Touschek Lifetime Improvement

(Some improvement and lengthening already occur from short-term wakefields)

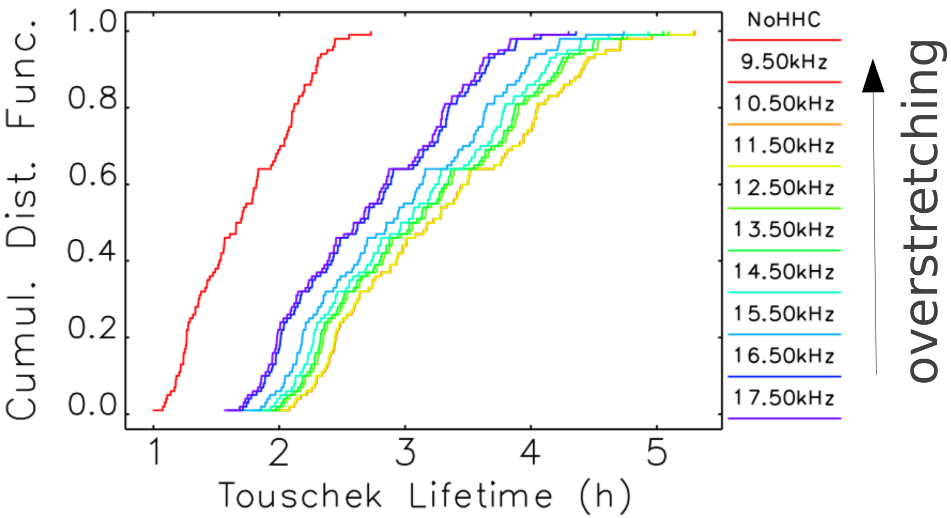


Figure 3.32. Distribution of Touschek lifetime for 100 ensembles for various detuning values of the HHC, assuming $Q_L = 6 \times 10^5$, compared to the lifetimes without the HHC, all for 48 bunches in 200 mA with $\kappa \approx 1$.

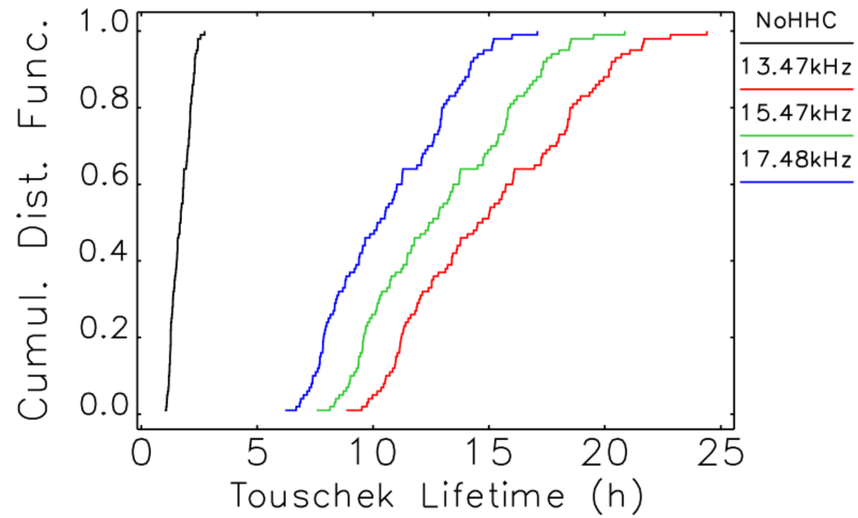


Figure 3.33. Distribution of Touschek lifetime for 100 ensembles for various detuning values of the HHC, assuming $Q_L = 6 \times 10^5$, compared to the lifetimes without the HHC, all for 324 bunches in 200 mA with $\kappa \approx 1$.