Coupling Definition Options for the SR

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This note discusses some options for computing the "coupling," which is a measure of the ratio of the vertical to horizontal emittance. The purpose is to communicate to x-ray users the options that are available and ideally agree on a single number that is the most useful to those who care about vertical emittance. This will help machine physicists to set up the machine in a consistent fashion prior to delivering beam.

In the simplest case, the emittance is just the product of the rms beam size and divergence. E.g., the horizontal emittance is

$$\epsilon_x = \sigma_x \sigma_{x'} \tag{1}$$

However, the spatial and angular coordinates of individual particles have contributions from both betatron motion and dispersion. Dispersion is the tendency of particles with higher or lower momentum to have different displacement and/or slope. Betatron motion is the fast oscillatory motion exhibited by all particles in the beam, which results in a non-zero beam size and divergence even for a hypothetical mono-energetic beam. We write the individual particle coordinates and slope as

$$x = x_\beta + \delta\eta_x \tag{2}$$

$$x' = x'_\beta + \delta\eta'_x \tag{3}$$

where primes indicate derivatives with respect to distance traveled, η is the dispersion function,and δ is the particle's fractional momentum offset.

We can compute the total, or effective, horizontal emittance ϵ_x from the definition

$$\epsilon_x = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}, \tag{4}$$

which reduces to Equation (1) when the beam ellipse is upright, i.e., when $\langle xx'\rangle = 0$. The betatron or natural horizontal emittance is

$$\epsilon_{x,\beta} = \sqrt{\langle x_\beta^2\rangle\langle x'^2_\beta\rangle - \langle x_\beta x'_\beta\rangle^2}. \tag{5}$$

Similar equations hold for the vertical plane.

For both the vertical and horizontal plane, then, we have two possible emittances to quote: the natural or betatron emittance, and the effective emittance. At locations where the dispersion and its slope are sufficiently close to zero, these are essentially the same. When the dispersion or its slope are nonzero, these may still be effectively the same depending if the user cares mostly about divergence or beamsize, respectively. In general, however, we have four possible ratios to use in characterizing the "coupling."

It turns out that the vertical dispersion is very nearly zero at ID straight sections, tending to have extrema midway between two straight sections and to have the form of a sinusoidal oscillation as a function of position. The slope of the vertical dispersion, however, is nonzero at the IDs. If the beam line optics image the beam, then the nonzero slope of the vertical dispersion may be somewhat irrelevant as it only adds to the divergence. In this case, the most relevant vertical emittance is the natural emittance.

If the beamline optics does not image the beam, then the vertical divergence is more relevant, since for a sufficiently long beamline without imaging, the divergence dominates. The dispersion slope may increase the $\sim 3\mu$rad vertical divergence by about 25% [1]. In that case, the relevant vertical emittance is apparently the effective emittance. However, for a "typical" APS undulator (U33) operated at 10 keV, the vertical divergence of the photon beam is dominated by the undulator contribution (about 5 μrad), which will diminish the impact of the effective emittance correction. This effect will be less significant at higher photon energies or for long undulators. For example, at 25 keV the divergence from the electron beam is about the same as the intrinsic divergence from the undulator.

On balance, for photon energies that are not "too high" and undulators that are not "too long," it appears that the natural vertical emittance is most likely to be of interest to users who care about vertical emittance.

In the horizontal plane, the dispersion $\eta_x \approx 0.17m$ is quite large at the IDs, contributing 163 μm in quadrature to the rms beamsize of 280 μm. The angular divergence due to dispersion slope resulting from residual errors in lattice correction is $\sim$1 μrad [1], which is an order of magnitude less than the divergence due to betatron motion. At 25m from the ID, the contributions to the x-ray beamsize from the electron beamsize and electron beam divergence are equal; after that, the divergence contribution begins to dominate. Only for a optics-free beamline of, $\sim$75 m length would the divergence terms completely dominate; in such a case, the relevant horizontal emittance is the natural emittance. However, this seems an unusual case and in general we expect that the effective horizontal emittance is relevant to APS beamlines.

As Sajaev reminds us, the situation is different for the BM lines, where the vertical dispersion is typically at the maximum value. For these beamlines, the intrinsic photon beam divergence is very large, i.e., $1/\gamma \approx 73\mu$rad. This is much larger than the few μrad of the beam for 1% coupling. Hence, if the beamline optics do not image the beam, the vertical electron beam properties are irrelevant. For the 1% coupling case, we find that the dispersion contribution to the vertical beamsize is much smaller than the beamsize and generally smaller than the variation in the beamsize from one beamline to another. One reason is that the vertical beta function is quite large at this location. Hence, the natural vertical emittance is a reasonable approximation to the emittance seen by BM users.

Our conclusion, then, is that the relevant ratio for most situations should be the natural vertical emittance divided by the effective horizontal emittance. We propose that when setting the coupling, we use this ratio.

References

[1] This conclusion is based on examination of data provided by V. Sajaev for a 1% coupling case.