

Bimorph Mirror Tuning

Bimorph mirrors, i.e. adaptable optics mirrors, are becoming a common optics element on synchrotron beamlines. **Once** properly tuned (meaning, once the proper settings for focusing have been established) they focus beautifully. However, said “**once**” may require some effort. These are not “plug and play” devices.

Main advantage: Unlike traditional mirrors with mechanical benders, where the user gets one (or, at most, two) “knob” to adjust the focusing, bimorph mirrors come with many (commonly 16, can be more) “knobs” to play with. Thus, many more degrees of freedom, to get things right.

Main problem: Lot’s of “knobs” to play with, many more ways to get things wrong.

The results presented here are from the ChemMatCARS beamline. Work mostly done by

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Background

During the decade 2000-2010 the ChemMatCARS (sector 15ID) beamline did operate using Si mirrors from Oxford (SESO). As part of an upgrade, in 2010, the mirrors were replaced by a new set, from Accel (SESO). This set includes:

1st mirror – 16-element bimorph, vertically focusing

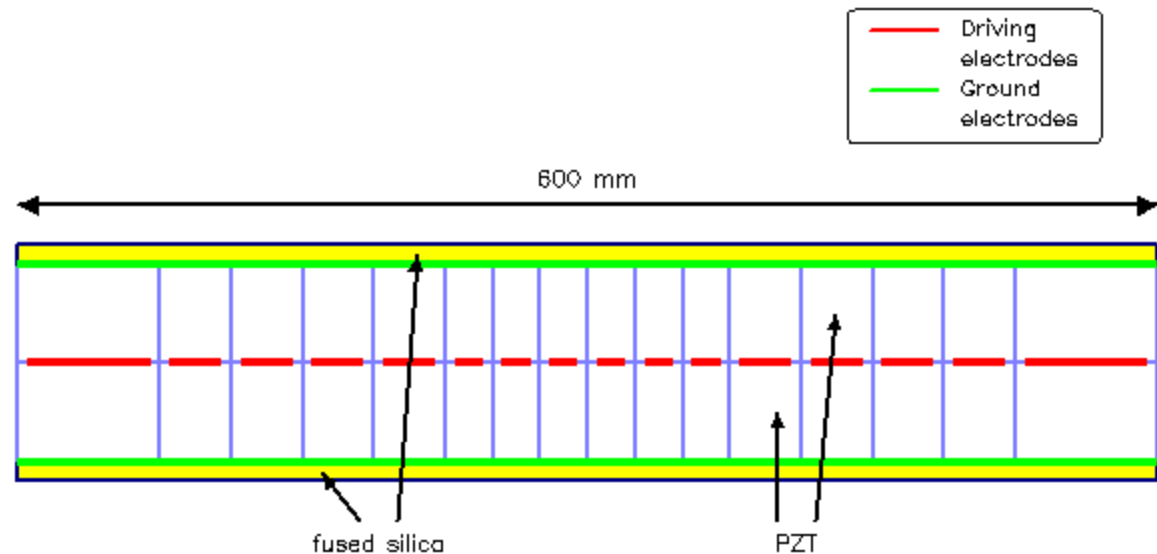
2nd mirror – Si flat

Slope errors 0.6-0.7 μrad , roughness 2.5 \AA (maybe☺)

(Both mirror sets have 3 stripes, bare fused silica (or Si), Rh and Pt)

Bimorph Mirror side view

Mirror located at
32.5m from source,
focusing to 48-62 m.



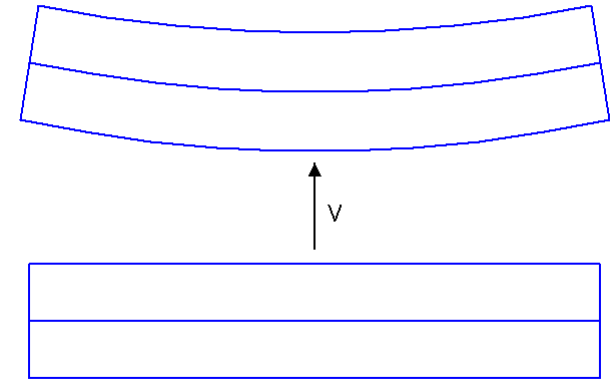
Applying voltage

Each mirror segment can be bent individually, by applying voltage

$$\text{Curvature} = \frac{1}{\text{Radius}} = \text{Const} * V$$

Fitting to the mirror test data, as supplied by Accel, yields

$$\text{Const} = -5.1 * 10^{-4} \left(\frac{\text{microradian}}{\text{mm} * \text{volt}} \right)$$



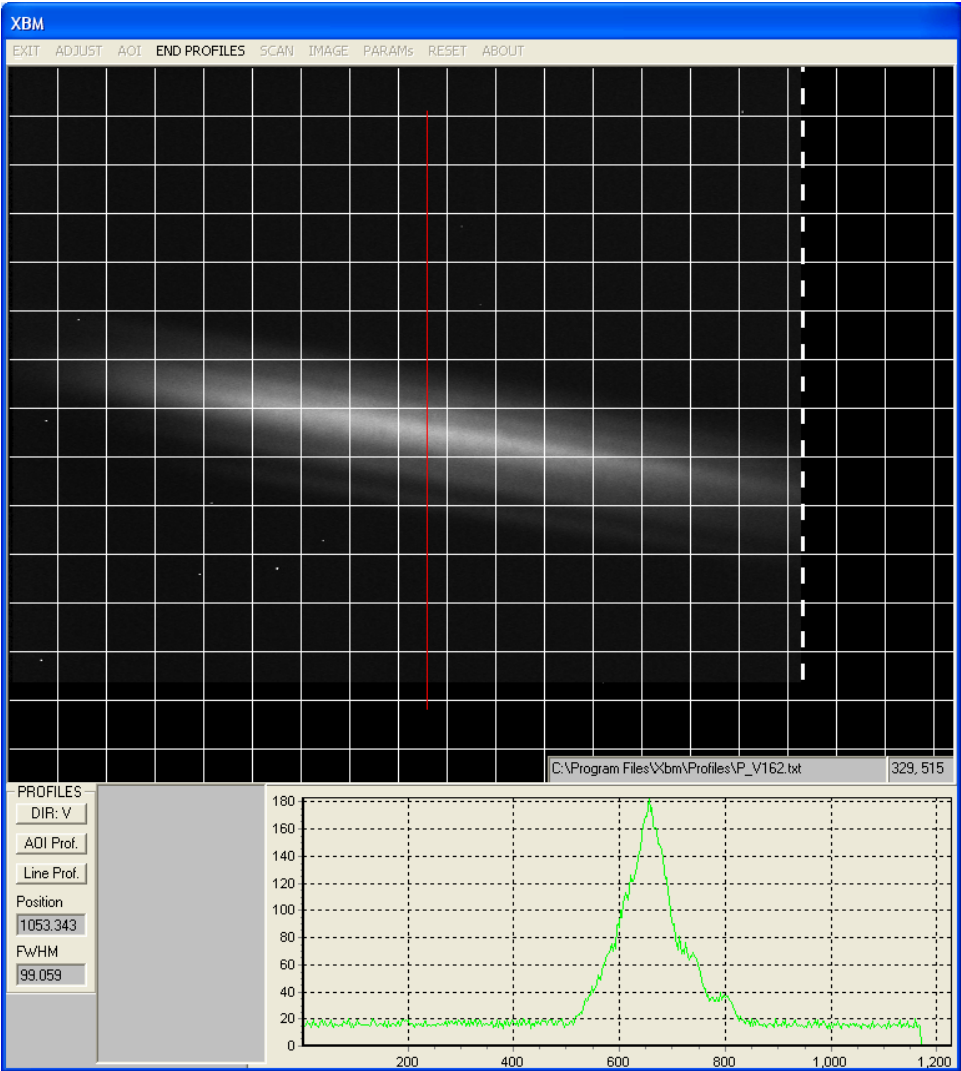
So, it is just a matter of applying the voltages required to bend the mirror to the proper shape. Only, there are 16 voltages! How to find them? Trial and error?

- Try raising all voltages by the same amount (for uniform bend). Get focusing, but not very good.
- Try tweaking individual voltages, one or two at a time. Still not very good. Sharper peak than what we could get with the old mirror, but lots of structure

So, the simpleminded approach doesn't work well (found out why, later). Time to get systematic.

Trial and error outcome

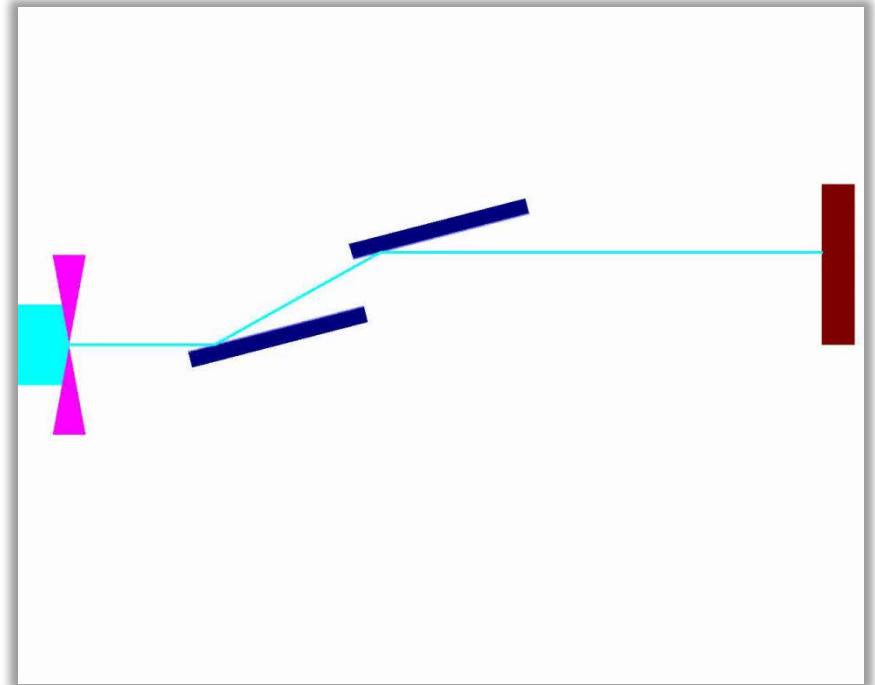
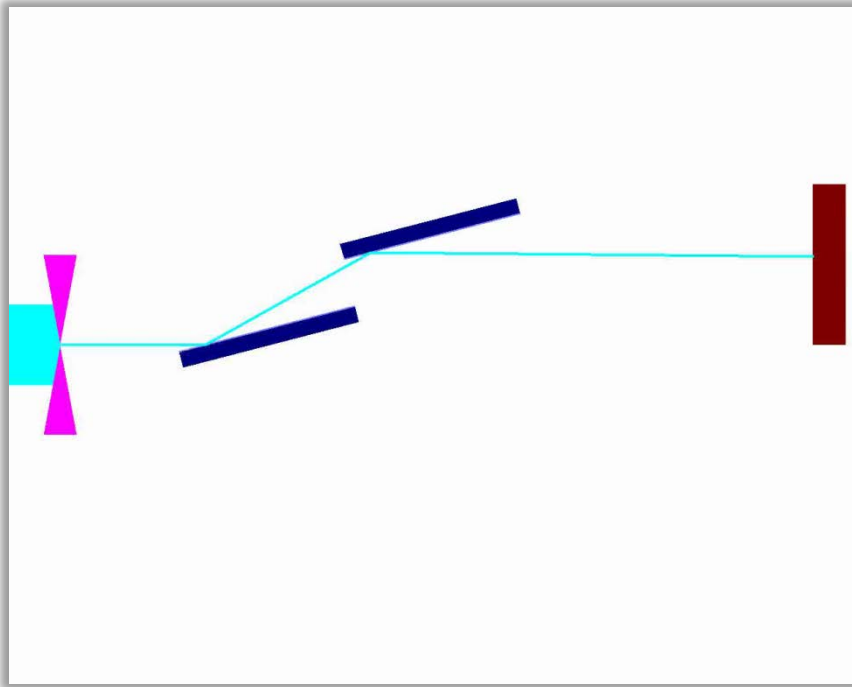
That's about the best before getting serious.



Mapping slopes

The mirror slopes can be mapped using a mirror scan:

$$\text{slope} = -\frac{y_{\text{det}}}{2 \times \text{distance}}$$



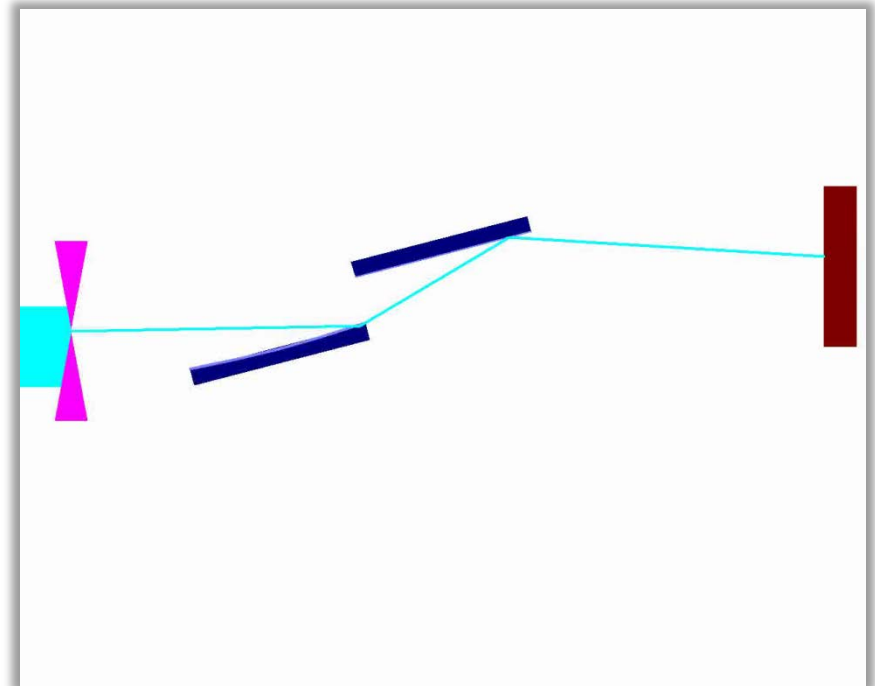
More mapping slopes

...or, alternatively, using a slit scan.
Mathematically, mirror scans and slit scans are equivalent. In reality, slit scans are preferable, being far less sensitive to positioning errors.

To minimize beam footprint on the mirror, the slit should be closed to

$$\approx \sqrt{L\lambda}$$

where L is the slit-mirror distance and λ is the wavelength being used. Closing the slit further than this will increase diffraction broadening.



A digression

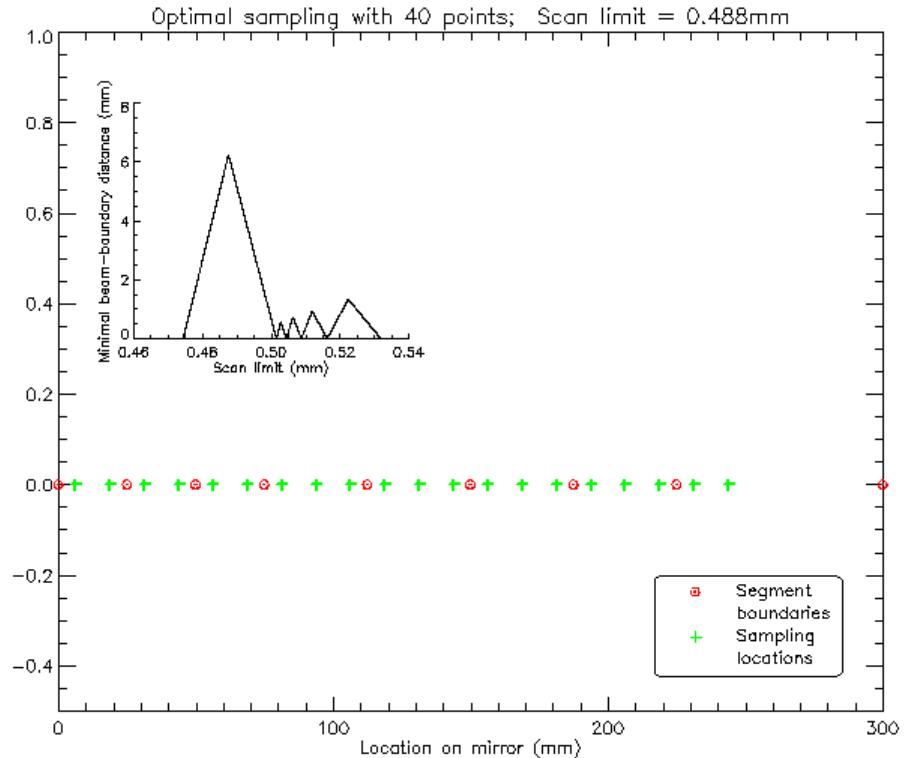
It is a good idea to choose sampling location so as to miss junctions between mirror elements. For constant step size it is a tad challenging, when the elements are not of constant length, but it can be done. For our mirror, at 2mr, the optimal scan is one of 40 points, with a step size (on the mirror) of 12.5mm. See on right.

The resulting parameters are:

Scan limit (vertical) = 0.488mm

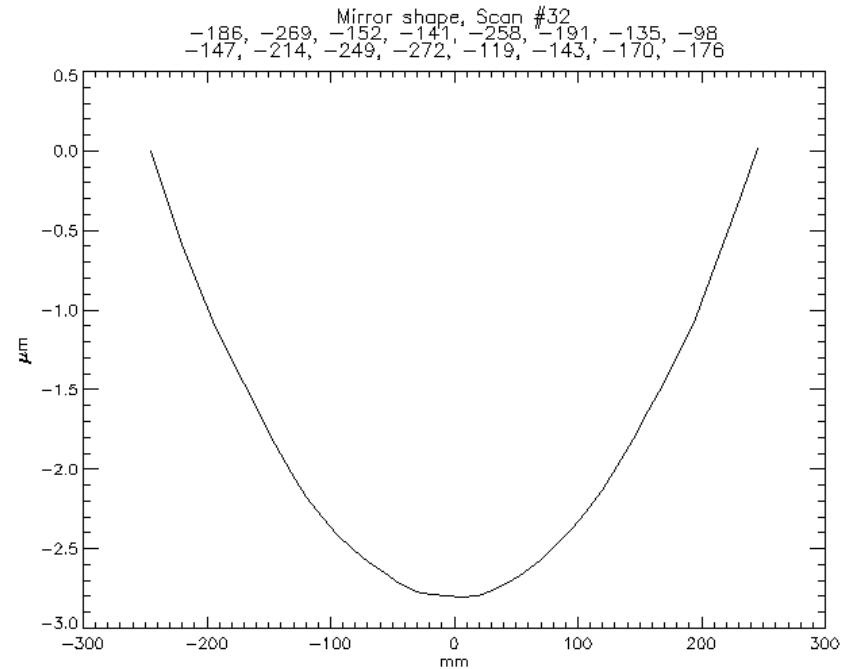
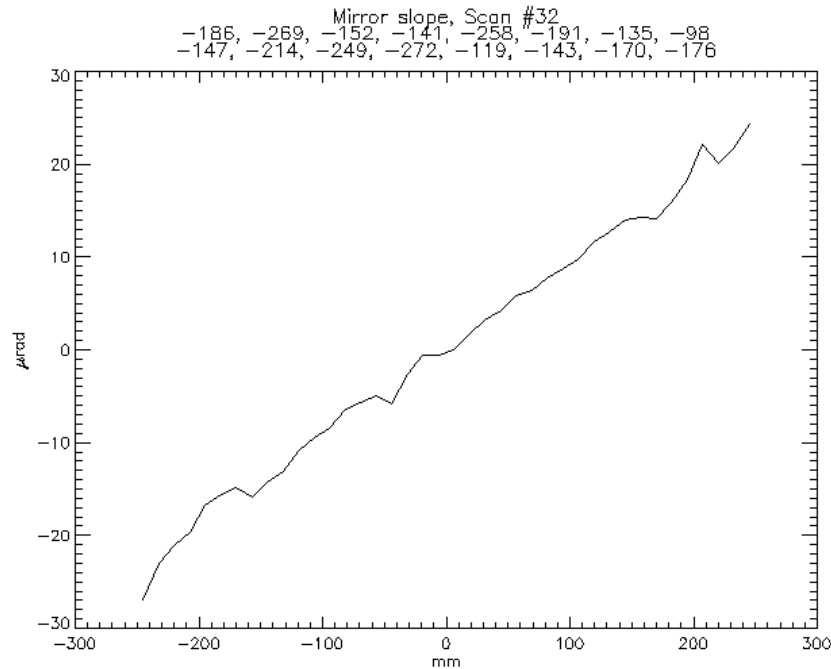
Scan step size = 12.5mm

Minimal boundary dist. = 6.247mm



Scan example

This was done with detector (Prosilica camera, resolution 1.96μ) at 48.5 m from source. The mirrors are at 32.1m (VFM) and 32.9m (VDM). The scan results correspond to radius of 10.5km and rms slope error (both mirrors, combined) of 1.0μ .



Matrix approach

The standard way to tune a bimorph is the matrix approach. We assume (for good reasons) linear response. Meaning

- The slope at each segment i ($i = 0, 1 \dots k-1$, where k is the number of segments) is constant within the segment, s_i .
- The slopes are combined into a vector \mathbf{s} , of dimension k .
- Similarly, the voltages applied to the segments form a vector \mathbf{v} , of same dimension.
- A linear relationship of the form $\Delta\mathbf{s} = \mathbf{M}\Delta\mathbf{v}$ is assumed, where \mathbf{M} is a constant (for a given mirror) matrix.
- Now, if the measured slopes are represented by \mathbf{s}_m while the theoretical slopes required for focusing are \mathbf{s}_f , then the voltage correction needed is $\Delta\mathbf{v} = \mathbf{M}^{-1}(\mathbf{s}_f - \mathbf{s}_m)$.

That's all nice, but it leads to two questions:

1. How does one find the matrix \mathbf{M} ?
2. Can it be inverted?

Matrix approach (2)

Evaluating \mathbf{M} :

1. Find the “base slope vector” \mathbf{s}_0 consisting of the slopes of all the segments when all the voltages are set to 0. Can either measure one slope per segment or measure a few and average them, in any case the result is a vector of length k (seg. number)
2. Similar to the above, find the slope vector $\mathbf{s}_1(\Delta v)$ which obtains with first segment at Δv and the rest at 0. Calculate the vector $\mathbf{m}_1 = (\mathbf{s}_1(\Delta v) - \mathbf{s}_0) / \Delta v$.
3. Repeat (2) for segments $2, \dots, k$, to generate the vectors $\mathbf{m}_2, \dots, \mathbf{m}_k$.
4. Pack the \mathbf{m} vectors together, creating the $k \times k$ matrix the columns of which are the vectors \mathbf{m}_i , i.e. $\mathbf{M} = (\mathbf{m}_1, \dots, \mathbf{m}_k)$. From its construction, for arbitrary voltage vector \mathbf{v} and the resulting slope vector \mathbf{s} we've $\mathbf{s} - \mathbf{s}_0 = \mathbf{M}\mathbf{v}$, thus $\mathbf{v} = \mathbf{M}^{-1}(\mathbf{s} - \mathbf{s}_0)$.

The process may appear tedious but, in principle, it needs to be done only once. But, turns out there is a problem. **The matrix is singular!**

This is not accidental. Mathematically it stems from the fact that any slope vector must satisfy the relation $\sum s_i l_i = \mathbf{s} \cdot \mathbf{l} = 0$, where l_i is the length of segment i and \mathbf{l} is the constant vector made of all the segment lengths. Thus, all the different \mathbf{s} vectors are not independent and the matrix made of them must, indeed, be singular.

In short, it is not an accidental bug, though it certainly is not a useful feature.

Matrix approach (3)

So:

- Strictly speaking, the inverse \mathbf{M}^{-1} doesn't exist, so it would appear that the matrix method cannot work.
- But, this problem may be circumvented by generating a “quasi-inverse”, using SVD. So, it would appear that the method may work after all.
- But, again, there is no good criterion for selecting a threshold for singular value rejection, and the result may greatly depend on this threshold. So, can it work?

Well, it can be made to work, in a fashion, but it is a bit of a lottery. Sometimes can get very good focus, other times, not so much. The method is not stable and a bit of experimental noise can throw it off.

The noise situation can be somewhat improved by oversampling. Instead of measuring slopes in just k locations (one per segment), we can use $l > k$ points (say 2,3,4 per segment). So, the \mathbf{s} vectors are of length l and the matrix \mathbf{M} (generated same as before) is $k \times l$. It is easy to show that in this case the previous formula for \mathbf{v} is modified to read

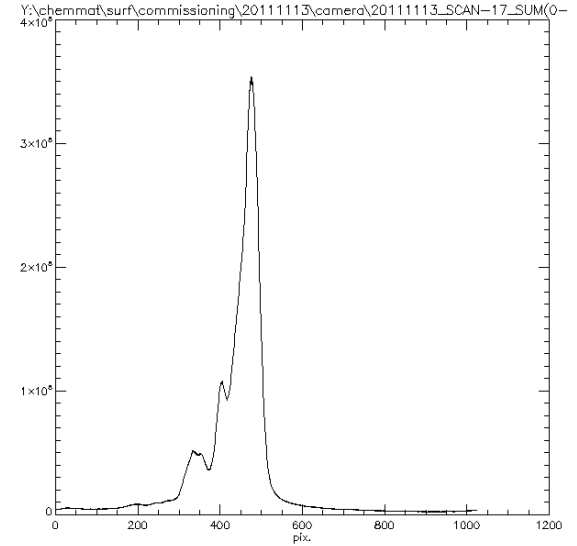
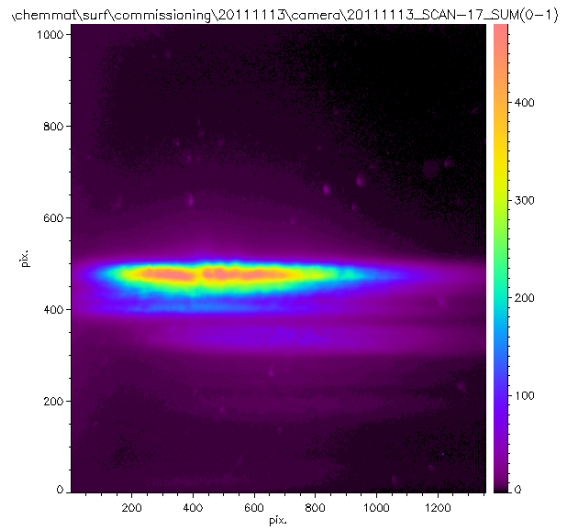
$$\mathbf{v} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T (\mathbf{s} - \mathbf{s}_0)$$

where \mathbf{M}^T is the transpose of \mathbf{M} . This may improve matters, but $(\mathbf{M}^T \mathbf{M})$ is still singular.

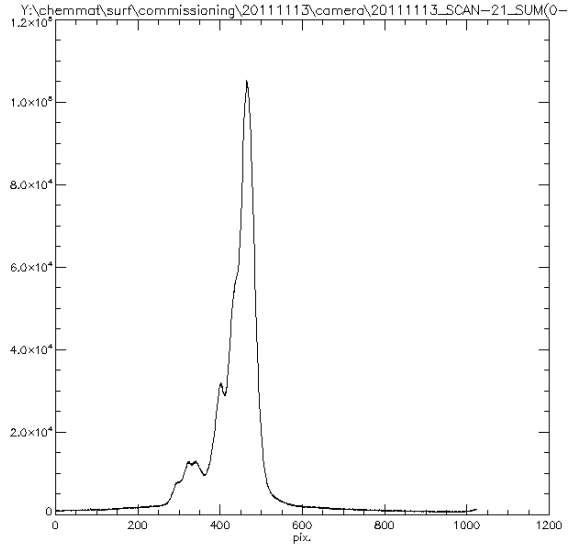
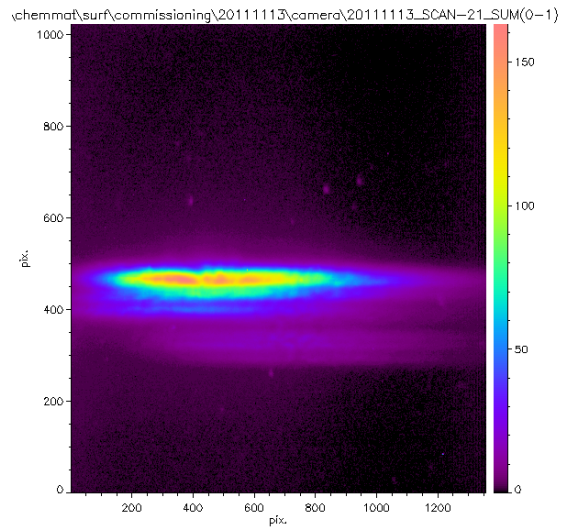
Some results

Using Prosilica camera, at 48.5m from source. 10keV beam.

After 2 tries.

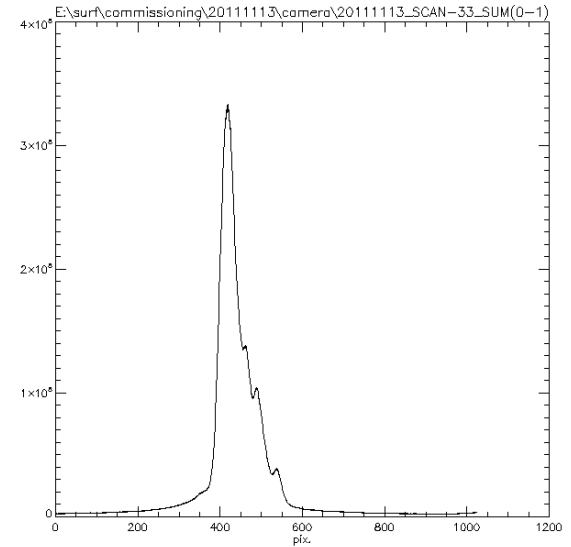
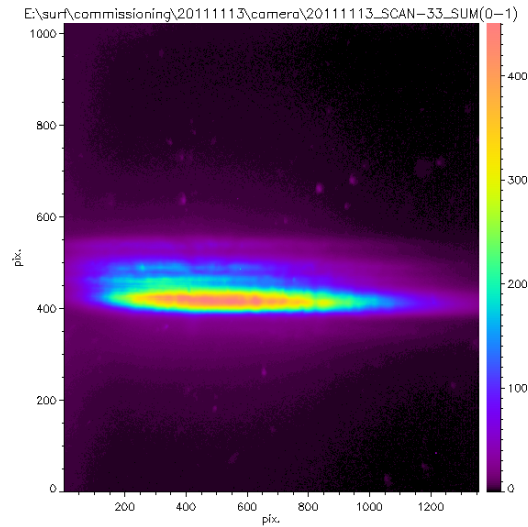


Another try

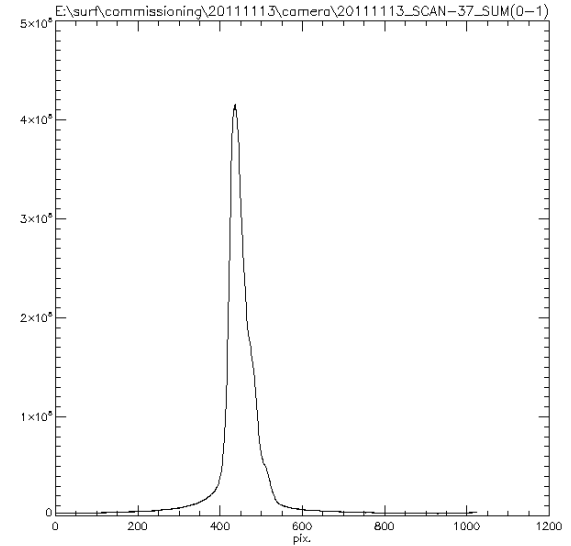
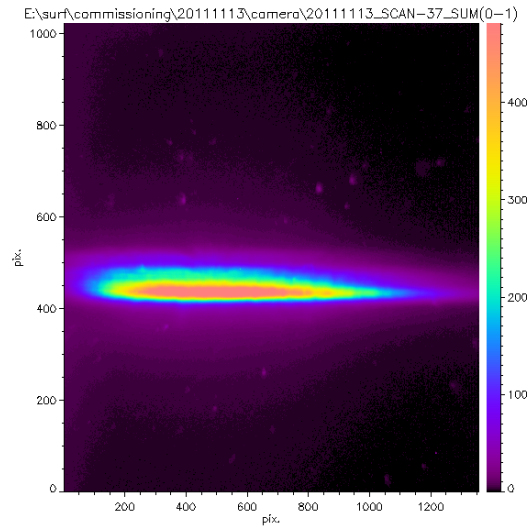


More results

Yet another try. Slightly better.



Some additional (non-matrix) tweaking. Pretty good, though some structure remains. FWHM about 90μ , though with tails. And, the next time around it was worse.



Something different – Fit function

The matrix approach may possibly work better, given sufficient effort. However, we decided to try something different, namely:

- Generate an easy to calculate function to model the bimorph mirror.
- The function should've parameters closely corresponding to the physical parameters of the mirror, i.e. segment voltages.
- Fit the function to any required mirror profile (or profile increment).

So, definition:

Given n intervals delimited by an ordered set of $n+1$ points, $x_0 < \dots < x_n$ and a set of n constants, $c_0 < \dots < c_{n-1}$, construct a function $F(x)$ with the following properties:

- Within each interval $[x_k, x_{k+1}]$ the function is given by $F(x) = a_k + b_k x + c_k \frac{x^2}{2}$, with c_k from the given set and a_k, b_k to be determined.
- $F(x)$ and $\frac{dF}{dx}$ are continuous across $[x_0, x_n]$.
- $F(x_0) = F(x_n) = 0$. This is not strictly necessary, but it is convenient and doesn't affect generality.

Fit Function (2)

Based on the definitions (i-iii) the function can be fully evaluated in terms of the parameters c_k alone. The result, within any interval $[x_k, x_{k+1}]$ is:

$$F(x) = c_k \frac{(x - x_k)^2}{2} + \frac{1}{x_n - x_0} \left((x_n - x) \sum_0^{k-1} c_j (x_0 - \bar{x}_j) \Delta x_j + (x_0 - x) \sum_k^{n-1} c_j (x_n - \bar{x}_j) \Delta x_j \right)$$

$$\text{where } \bar{x}_j = \frac{x_{j+1} + x_j}{2} \quad ; \quad \Delta x_j = x_{j+1} - x_j$$

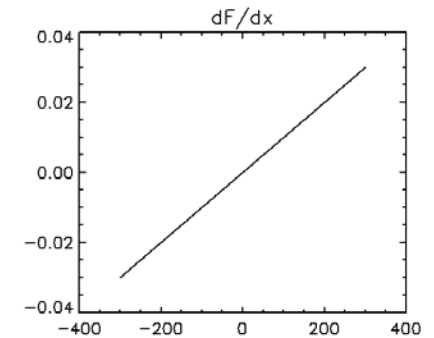
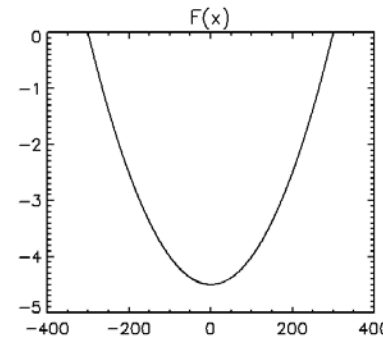
What's actually of more interest, for the fitting, is the derivative of $F(x)$, given by

$$F'(x) = c_k (x - x_k) - \frac{1}{x_n - x_0} \left(\sum_0^{k-1} c_j (x_0 - \bar{x}_j) \Delta x_j + \sum_k^{n-1} c_j (x_n - \bar{x}_j) \Delta x_j \right)$$

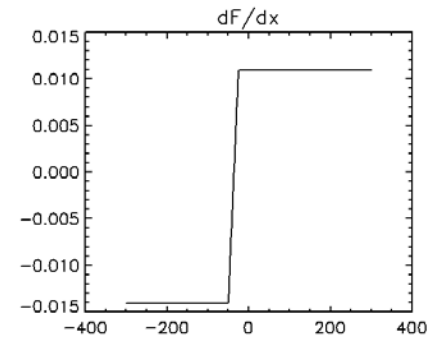
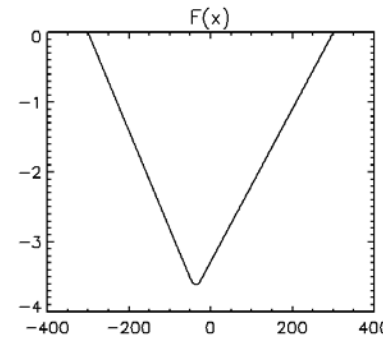
This can be used to fit to any required slope profile. The fit parameters are the c_k , of course. They represent local curvatures, thus are proportional to the segment voltages.

Fit function (3)

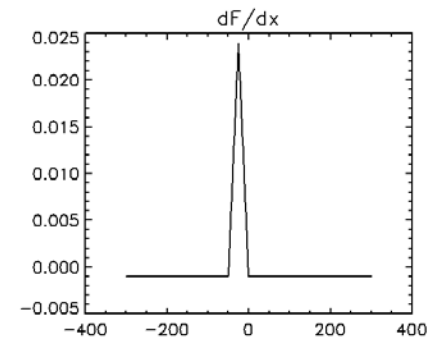
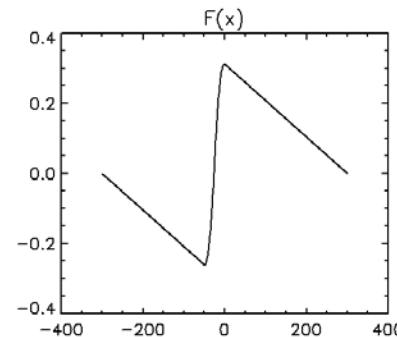
It is of some interest to see the shape of $F(x)$ and its derivative for various settings of the c_k values. The case on right corresponds to all the c_k being the same.



This to a single nonzero c_k



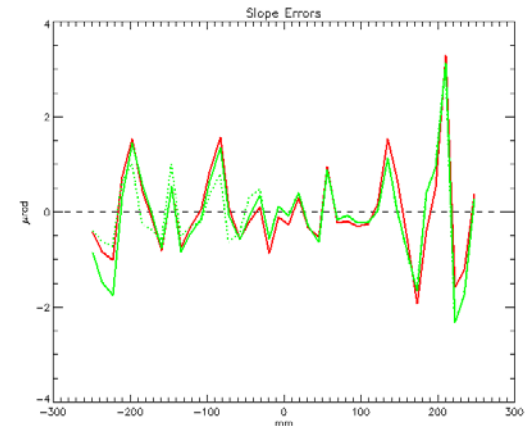
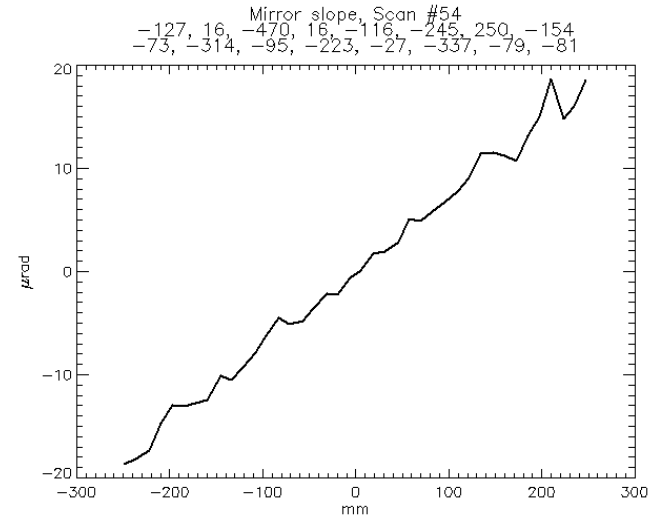
And this to two adjacent curvatures with same value and opposite signs.



Applying the fit

Focusing to 56m.
Mapping the slope:

Feeding the result to the fitting routine, the result is a new set of voltages. Estimated rms slope error $\sim 0.6\mu\text{r}$. Note, this is the **combined slope error of both mirrors**.

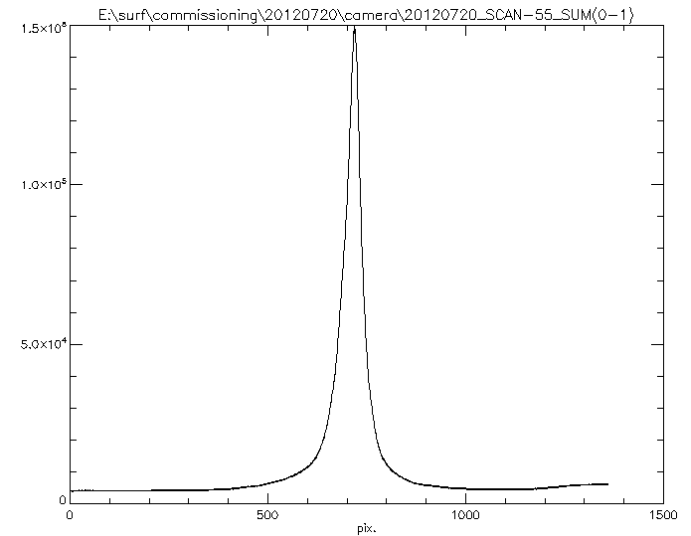
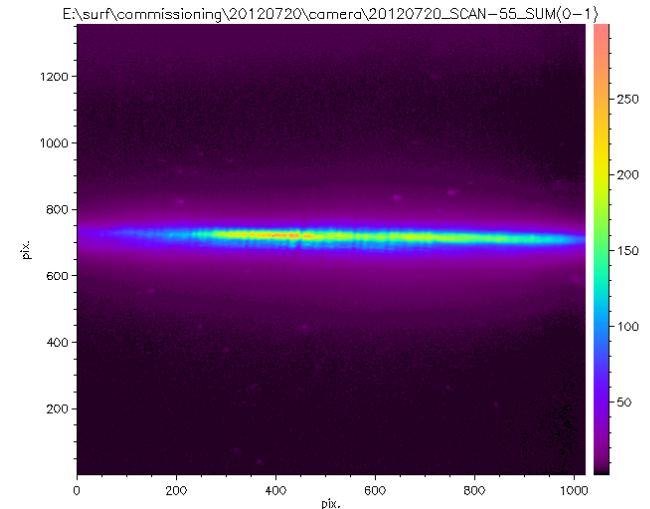


...And the outcome is

Lovely peak, sharp, no structure.

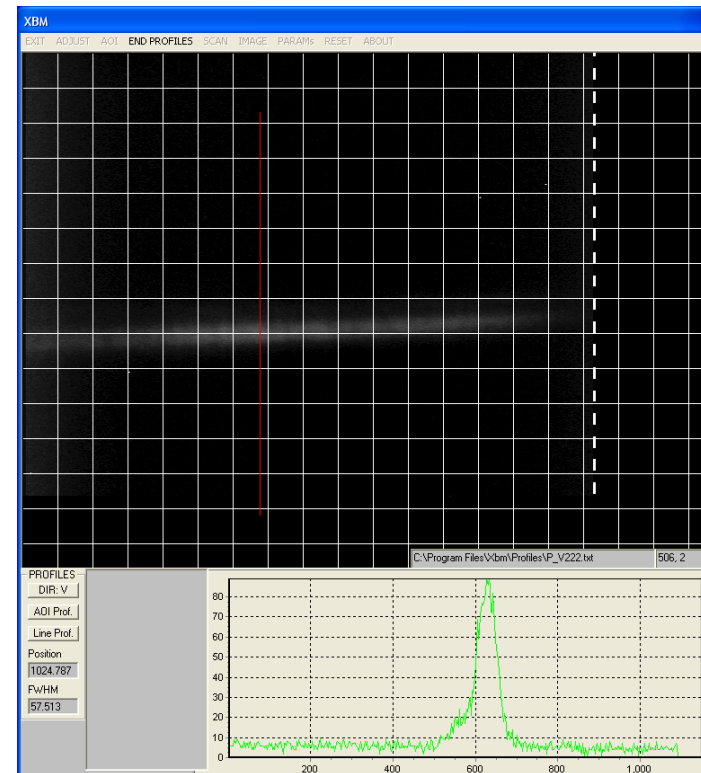
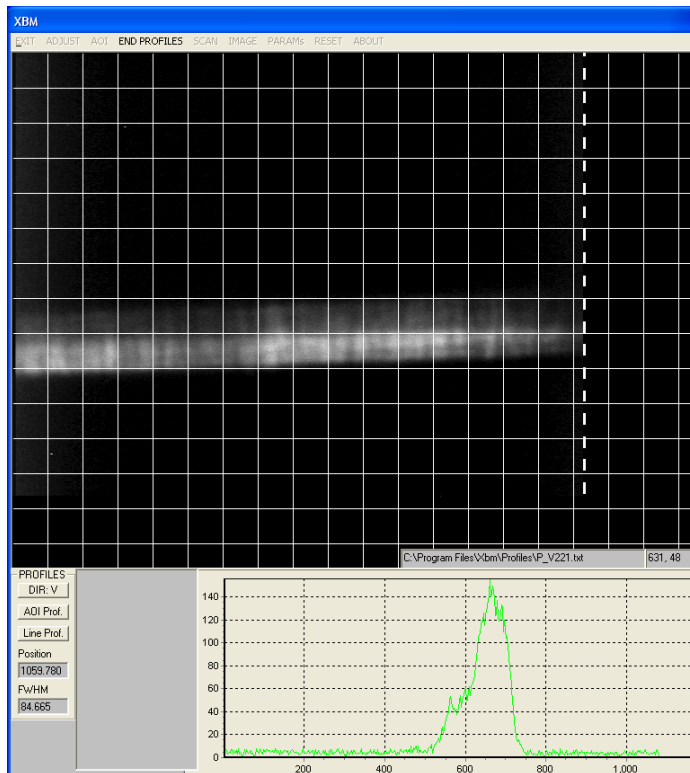
FWHM of $\sim 94\mu\text{r}$, but it is even smaller, since the camera is tilted. After correcting for the tilt we get

FWHM = $0.66\mu\text{r}$, agreeing with the fit estimate.



By the way...

A valuable feature of a bimorph is its ability to correct for flaws in other optical elements, in our case correct slope errors of the second mirror. This correction, though, necessitates maintaining constant spatial alignment of the elements. Below, on left, is the beam profile after the first mirror was incidentally vertically misplaced by 150μ . On the right is the profile after returning it to position, **with no change in focusing**.



Not all roses

Focusing of the silica stripe not as good as of the Rh and Pt stripes. Visual inspection indicates structure. Radiation damage? If anybody knows of something related, we'll be glad to hear.

